DE LA RECHERCHE À L'INDUSTRIE

# TOWARDS AN AUTOMATIC AND RELIABLE HEXAHEDRAL MESHING

Presentation using some illustrations from ***S. Owen***, Sandia National Laboratories, Albuquerque, USA

Tetrahedron V, Liège, July 2016

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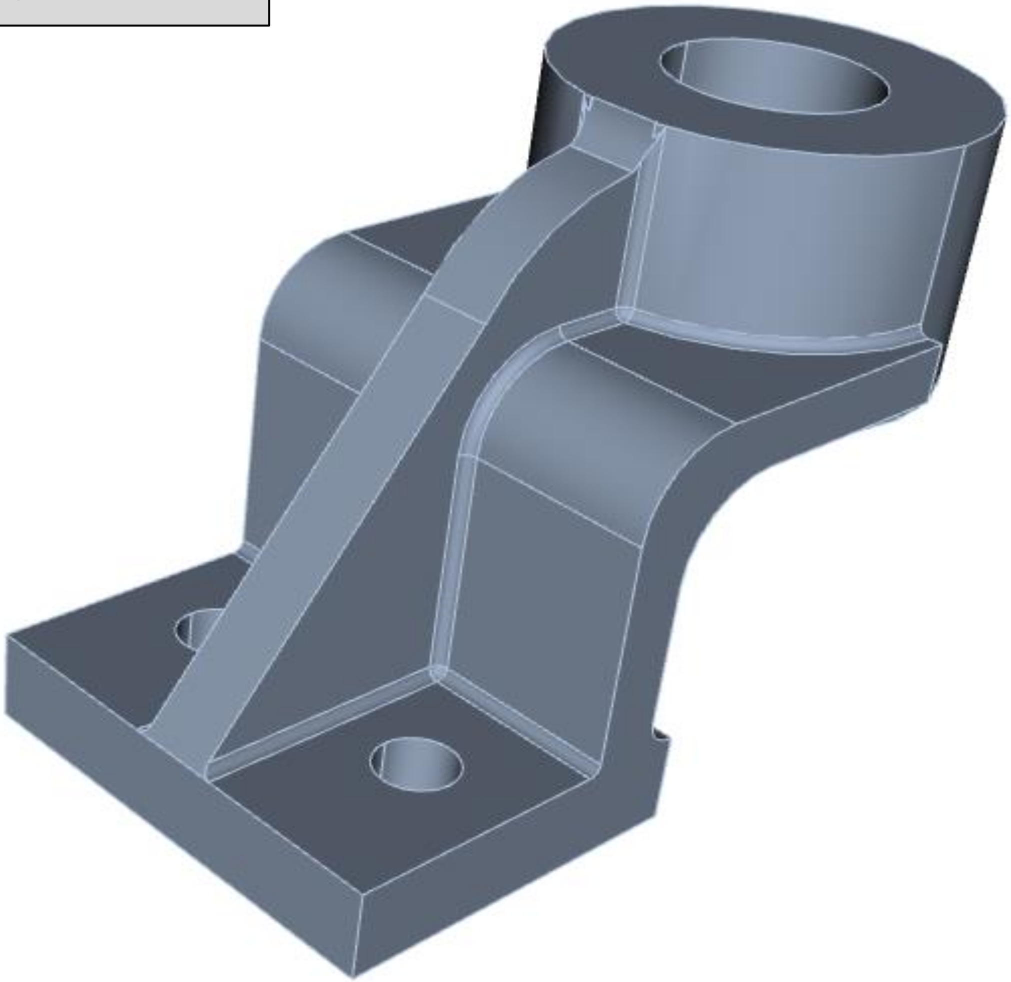
# INITIAL QUESTION

Let Ω be a CAD geometric domain, i.e. an assembly of BRep volumes, can we generate a hexahedral mesh that discretizes Ω?



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## A POSSIBLE SOLUTION

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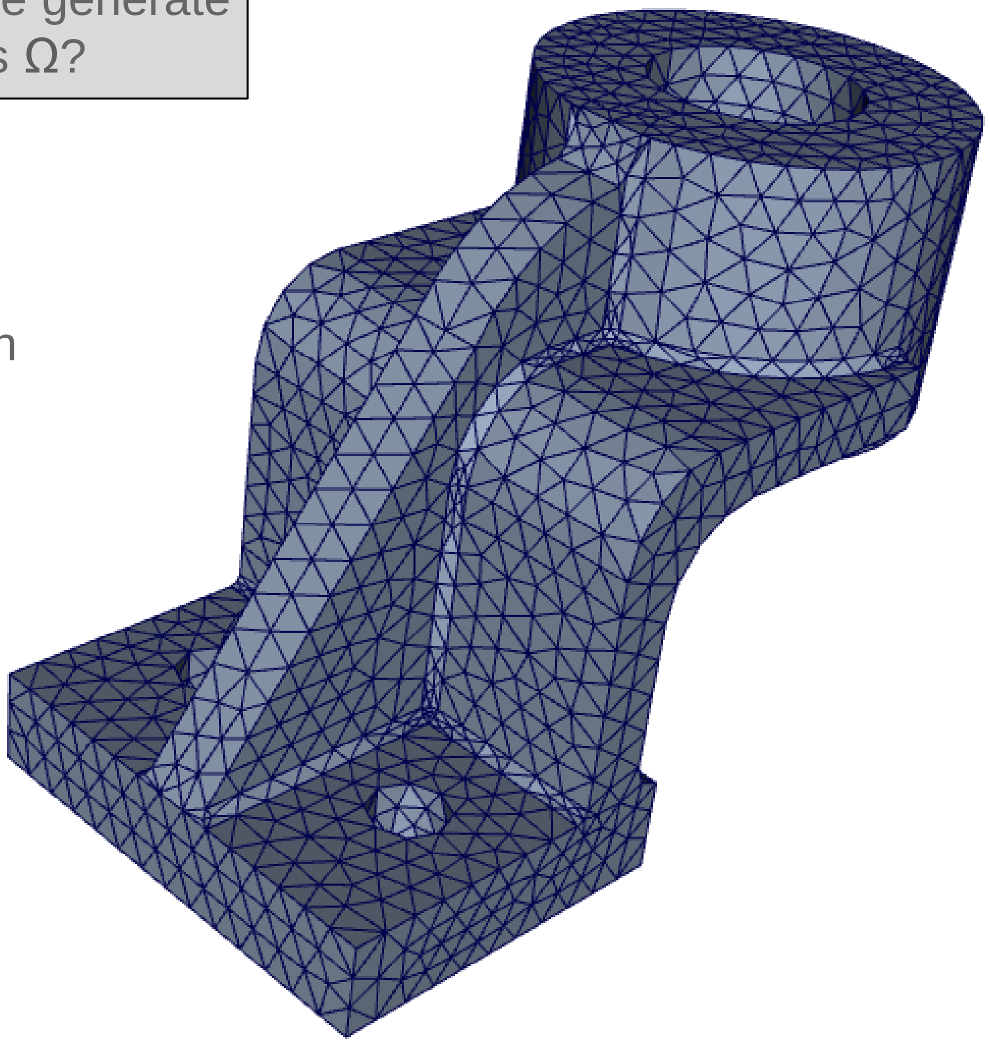

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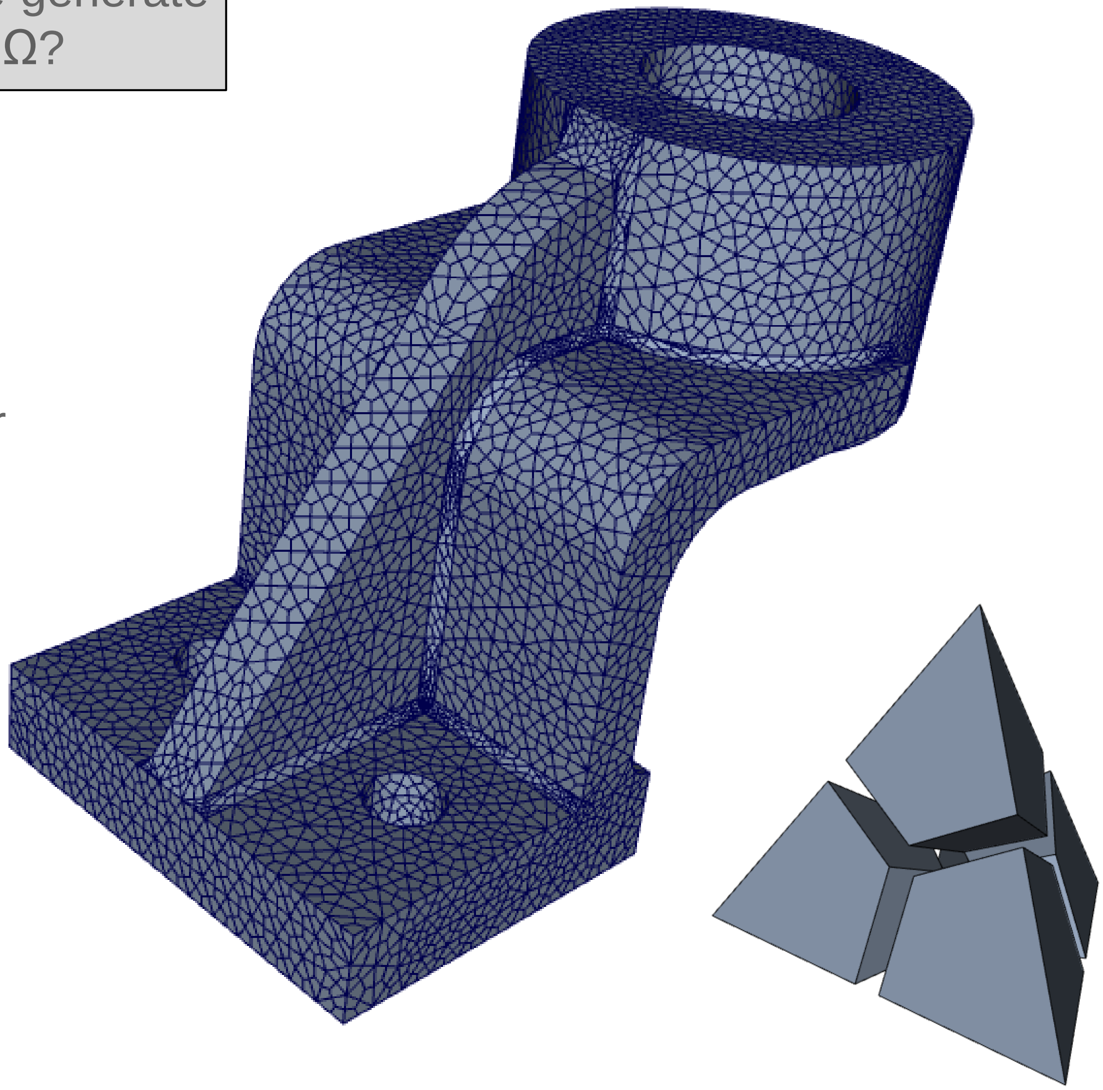
First, generating a tetrahedral mesh

Then split each tetrahedron into four hexahedral elements



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## A POSSIBLE SOLUTION

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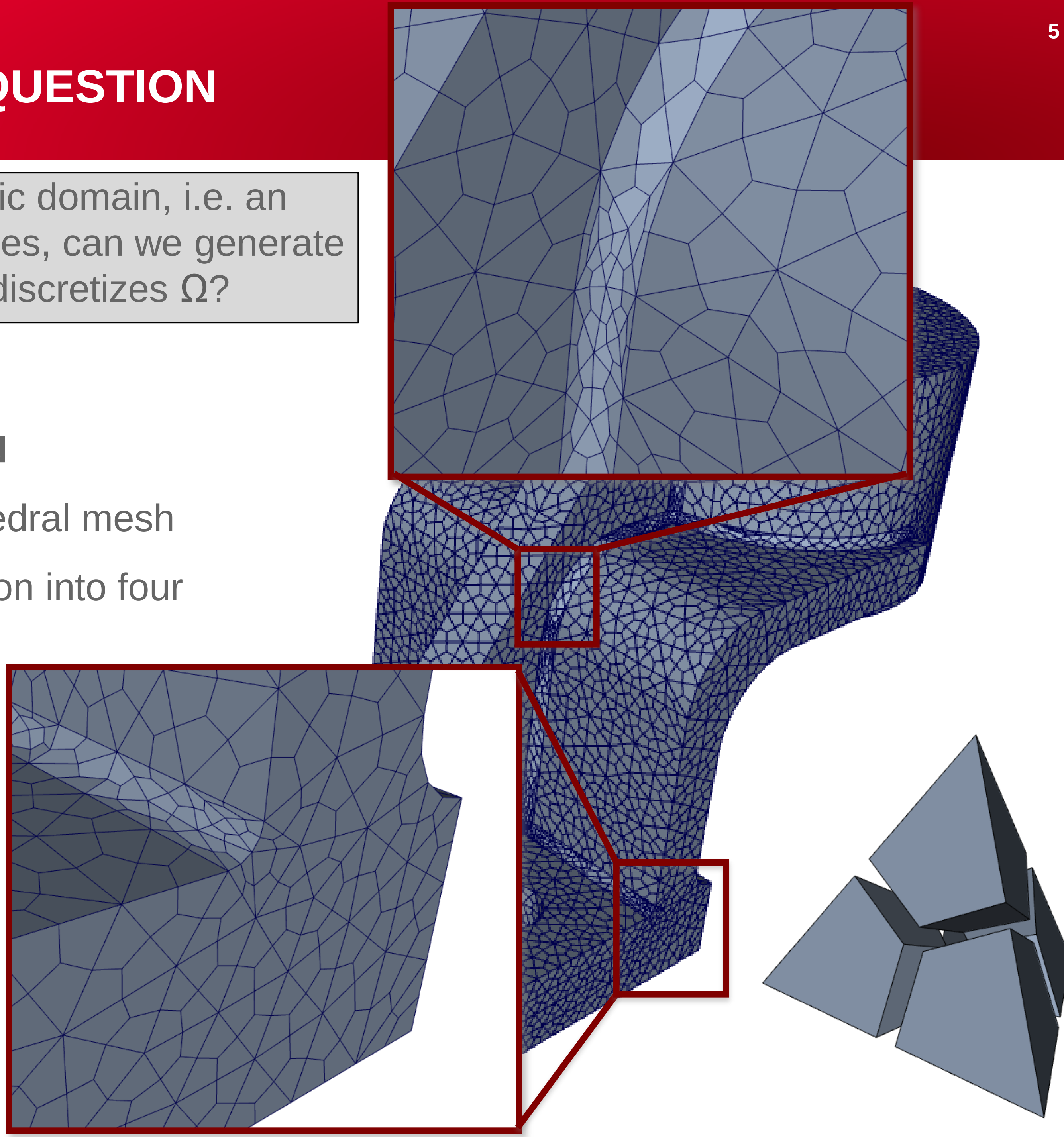
Then split each tetrahedron into four hexahedral elements

**BUT**

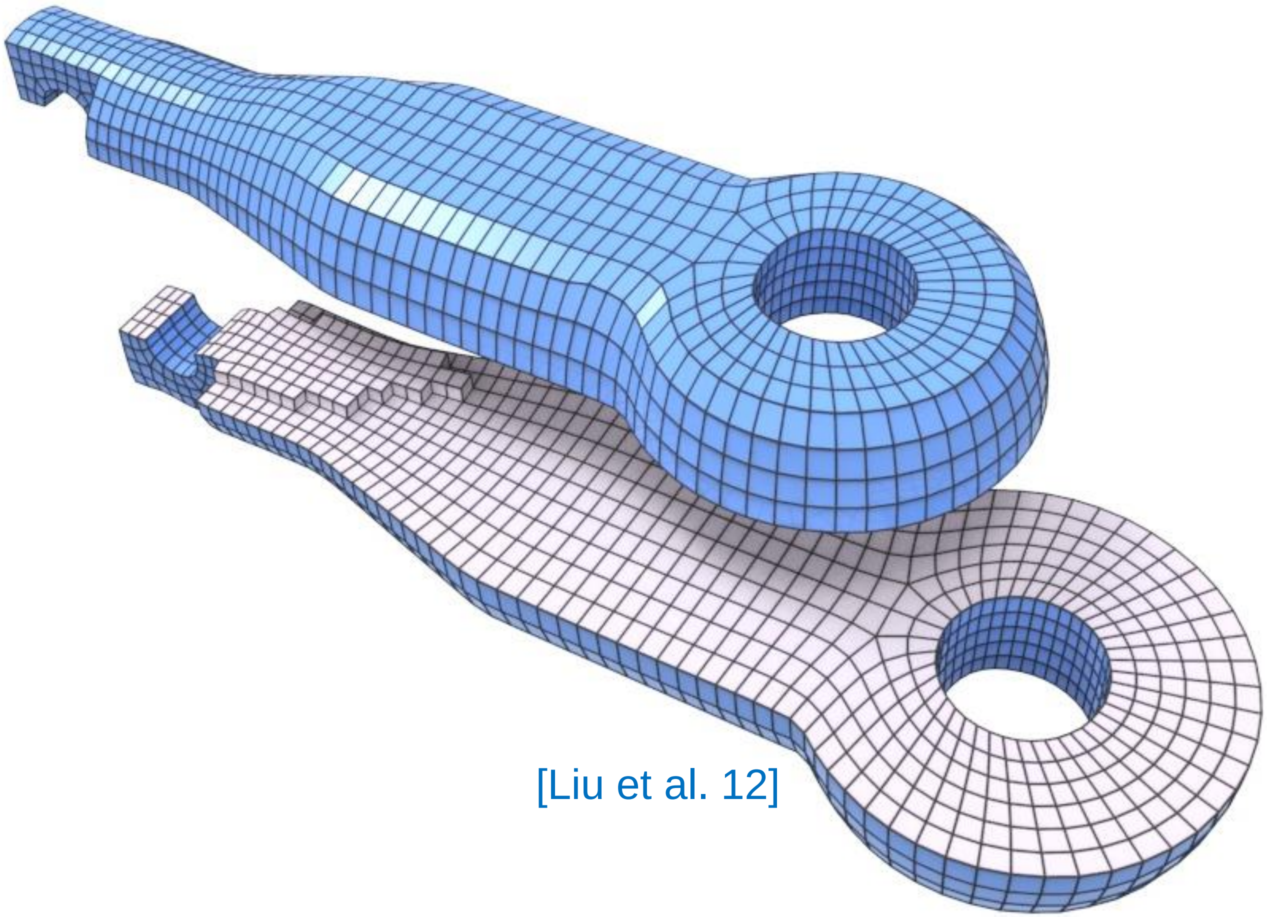
Generates bad quality elements

Unstructured mesh



# EXPECTED FEATURES (IN MOST CASES)

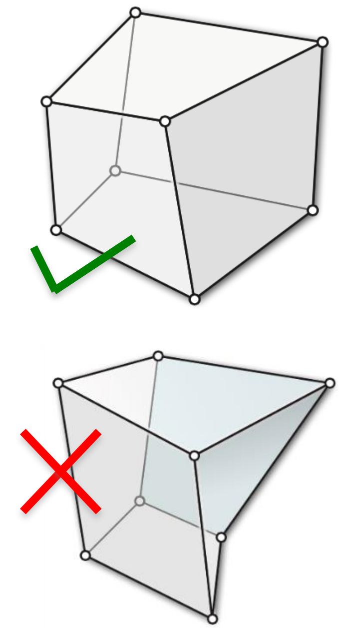
- Structure

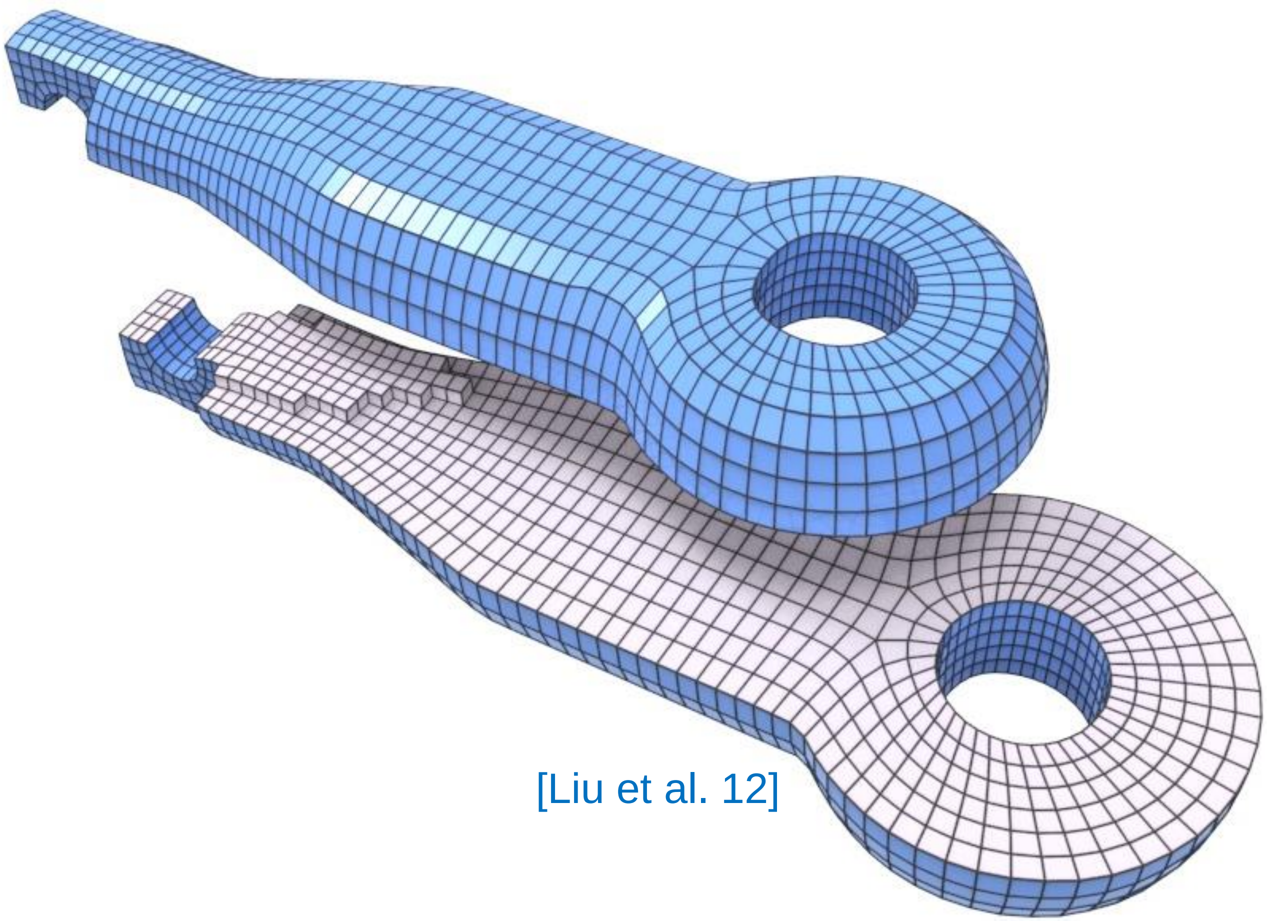


[Liu et al. 12]

# EXPECTED FEATURES (IN MOST CASES)

- Structure
- Low distortion of the cells





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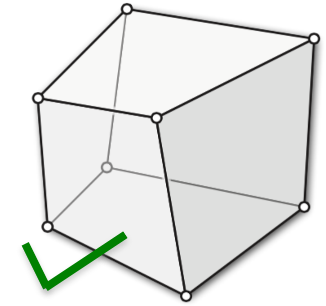
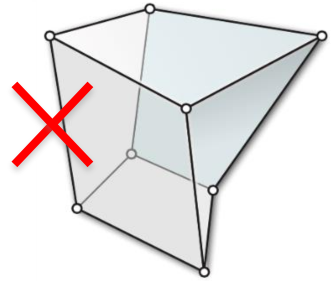
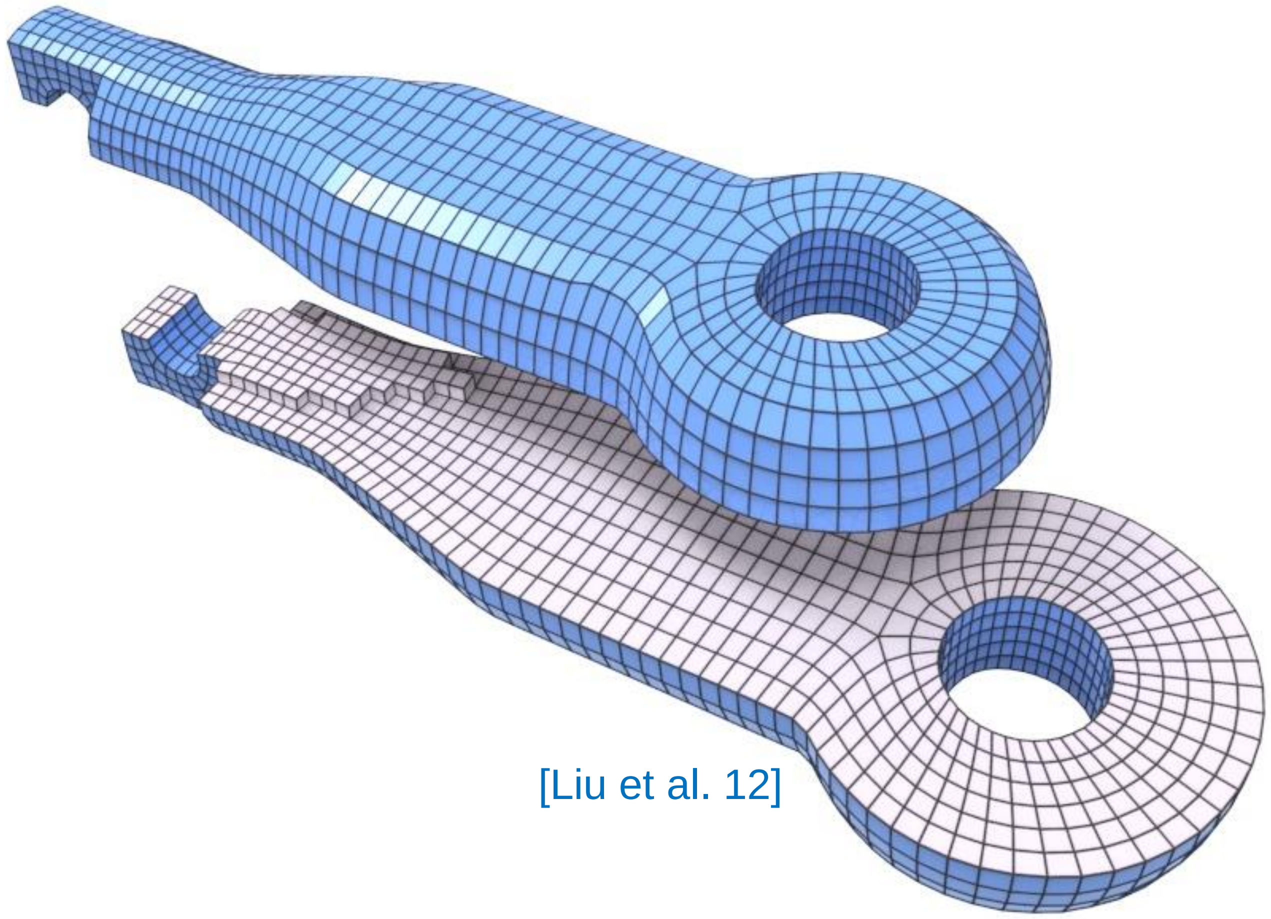
# EXPECTED FEATURES (IN MOST CASES)

- Structure
- Low distortion of the cells
- Geometric boundary alignment







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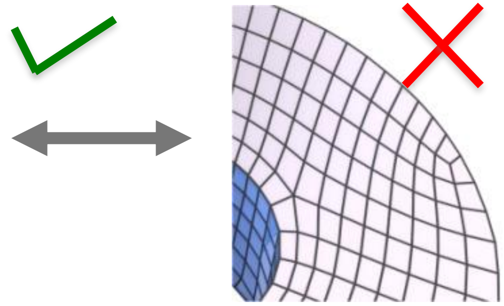
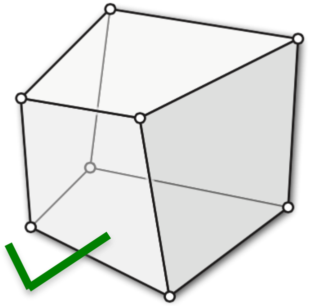
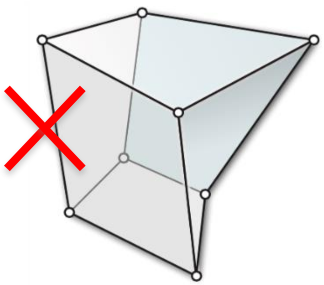
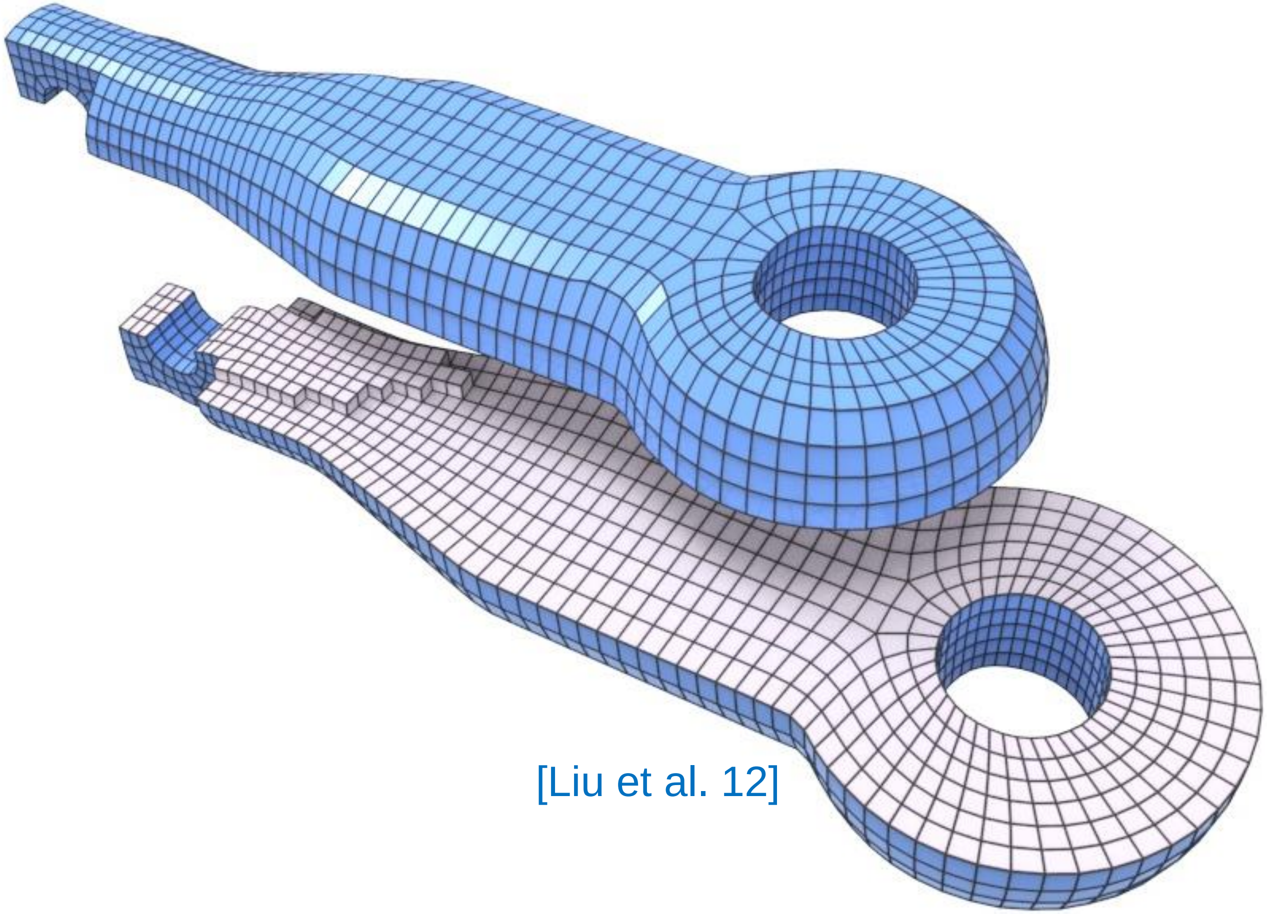


# EXPECTED FEATURES (IN MOST CASES)

- Structure
- Low distortion of the cells
- Geometric boundary alignment
- Size constraint







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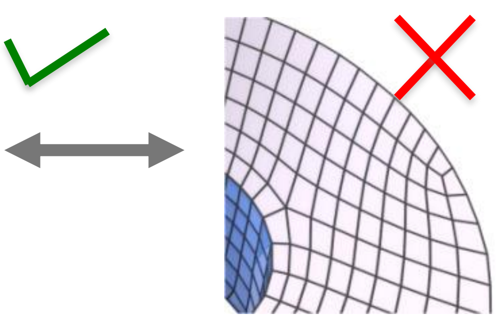
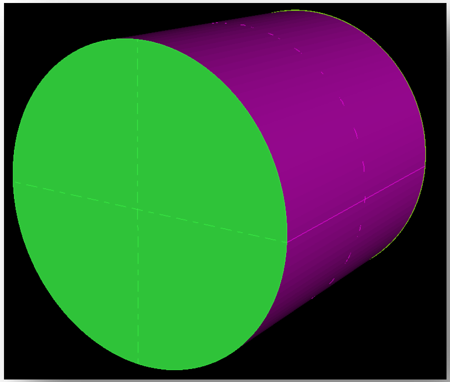
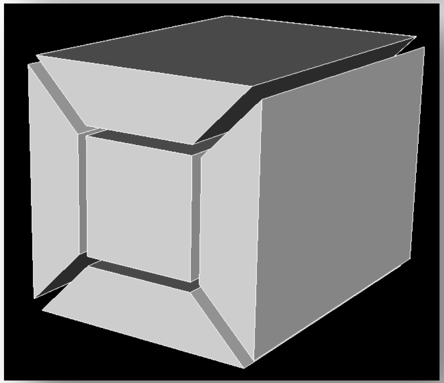
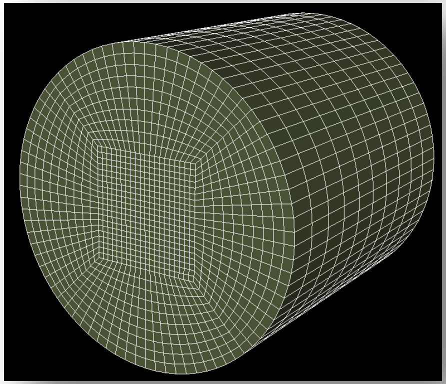


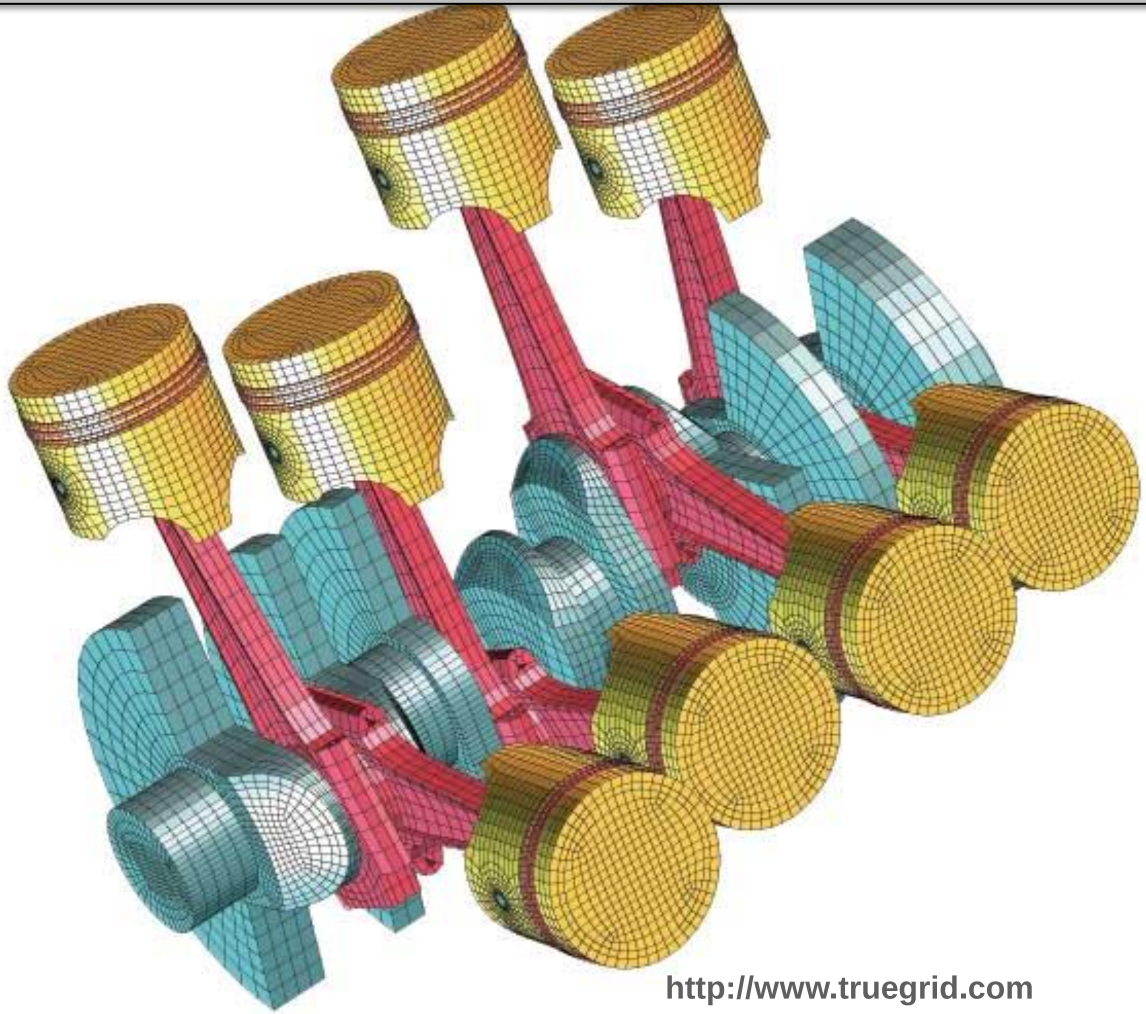
# SO, WHAT'S THE GOOD QUESTION?







Can we generate block-structured meshes that are aligned along simulation features and « anisotropic » ?



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# PLAN

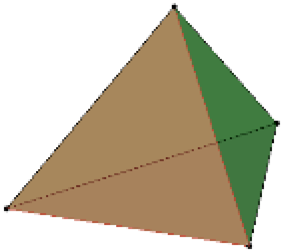
Why is it so difficult to generate full hexahedral meshes?

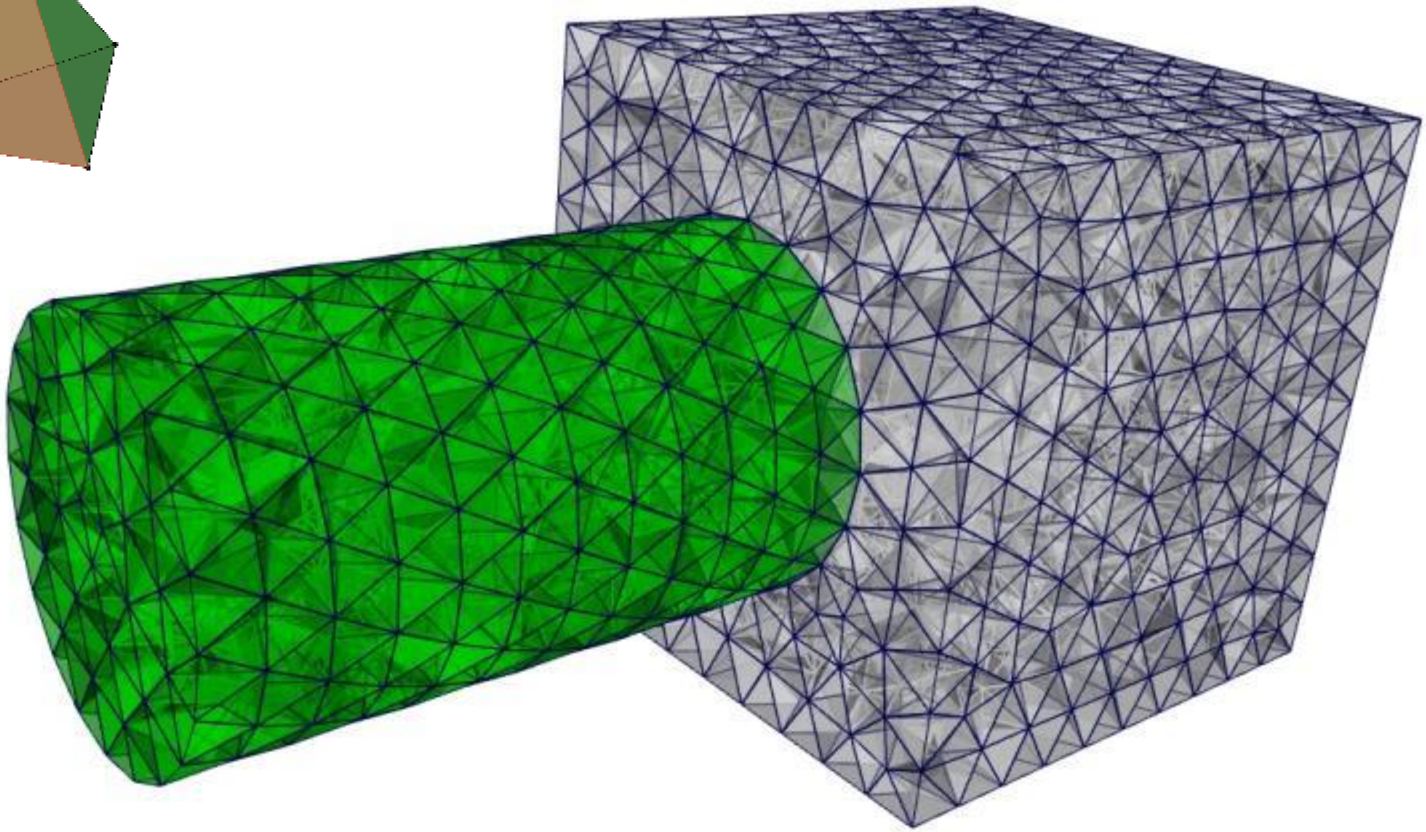
Which families of algorithms?

- Sweeping
- Geometric / Topological advancing-front approaches
- Medial axis
- Cartesian idealization –Submapping and polycubes
- Frame fields

# MESH GENERATION – BOTTOM-UP APPROACH

Vertices → Curves → Surfaces → Volumes





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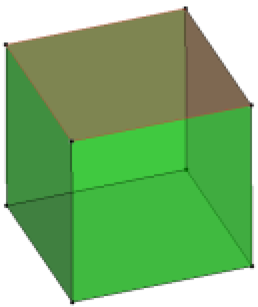

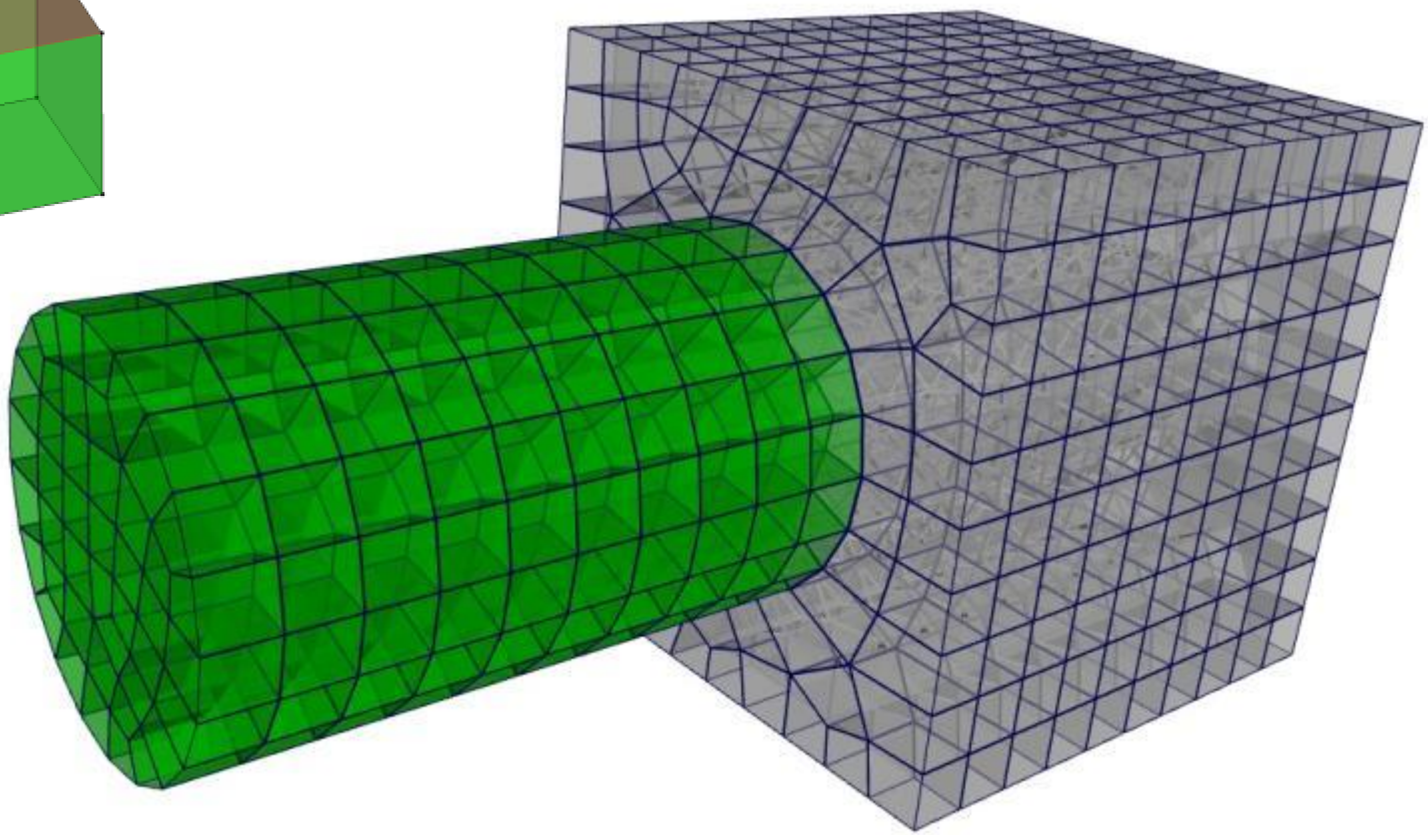

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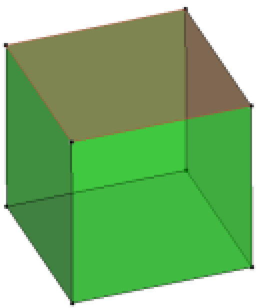
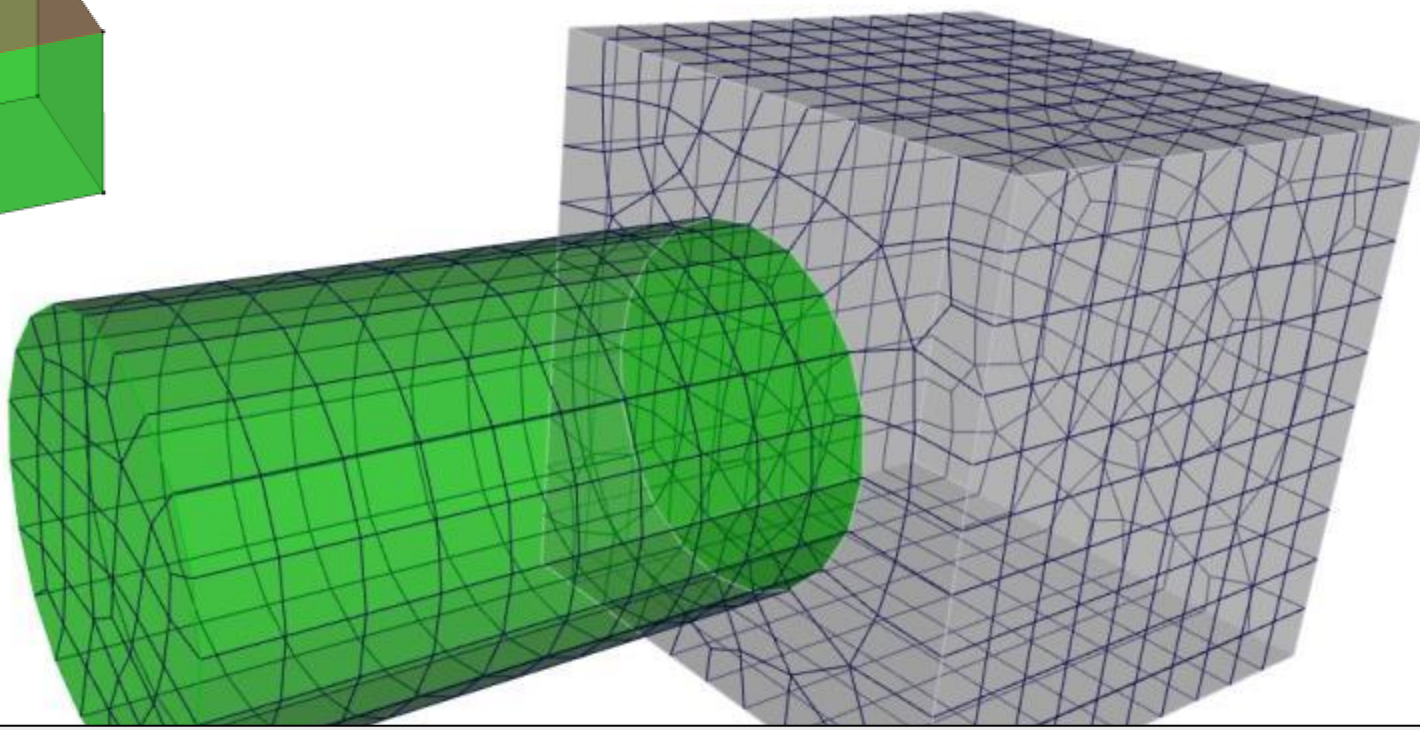




**Yes, I forgot to tell about something…**

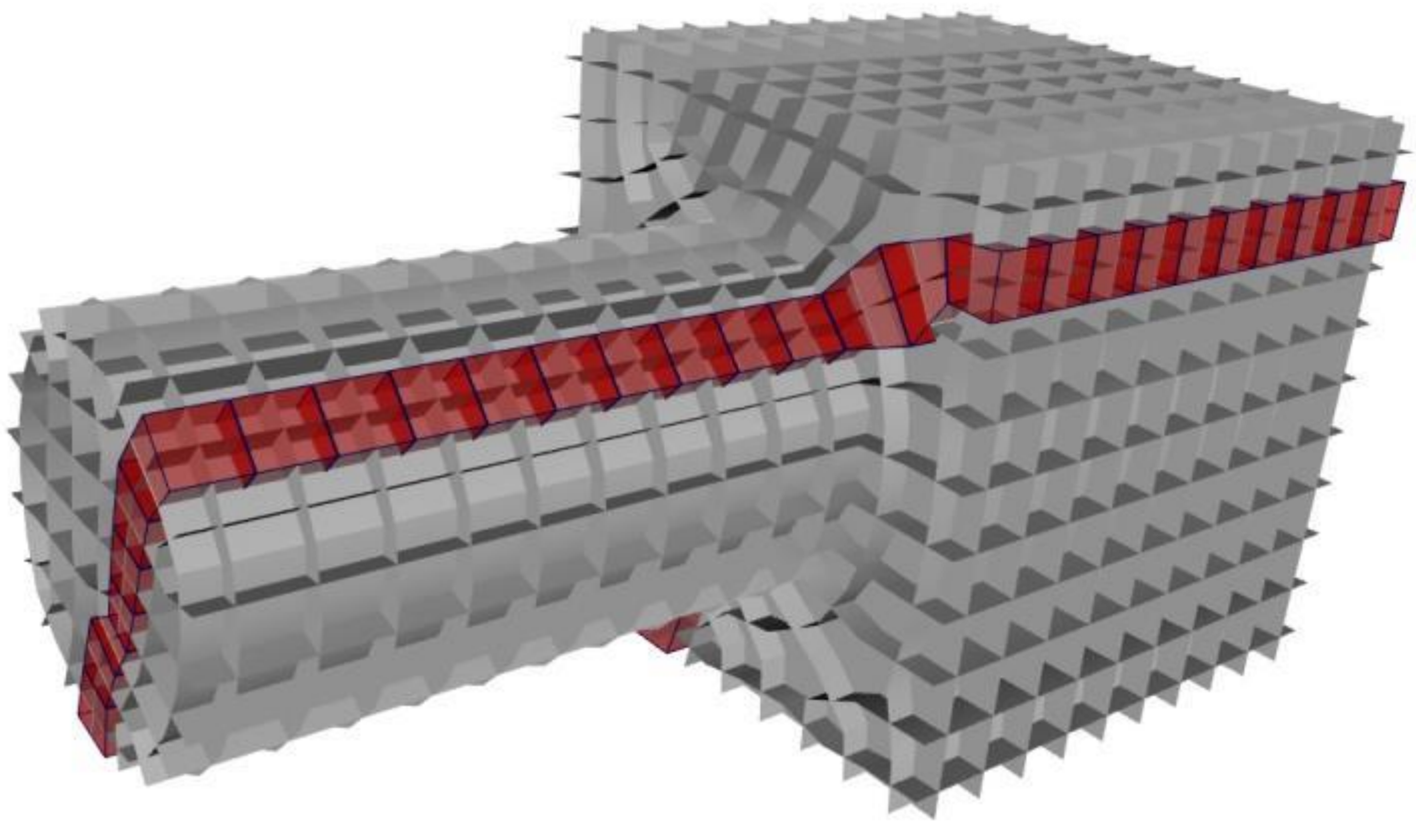
- Generating an hex. mesh for a volume enclosed by a quad mesh is still an open issue
- Known result: a **topological** solution exists if the number of quads is even

[S. Mitchell 95][B. Thurston 93]

# PROPERTIES OF HEXAHEDRAL MESHES

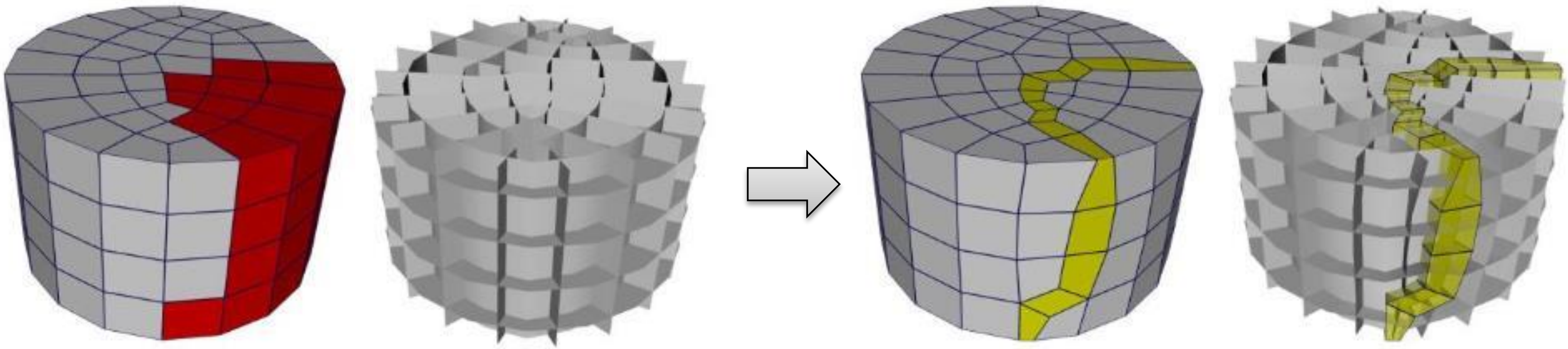
The dual of a hexahedral mesh is a simple arrangement of surfaces

[T. TAUTGES AND S. KNOOP, 2003]



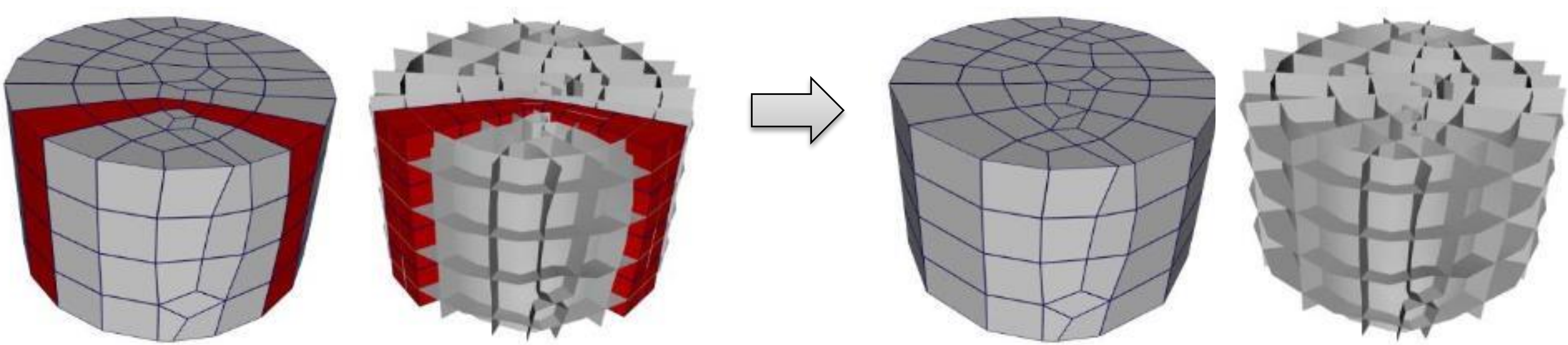
# Reliable operations to modify hexahedral meshes

## Sheet insertion



S. A. Mitchell and T. J. Tautges, *Pillowing Doublets: Refining a Mesh to Ensure That Faces Share At Most One Edge,* proc. of the 4th IMR, pages 231-240, 1995

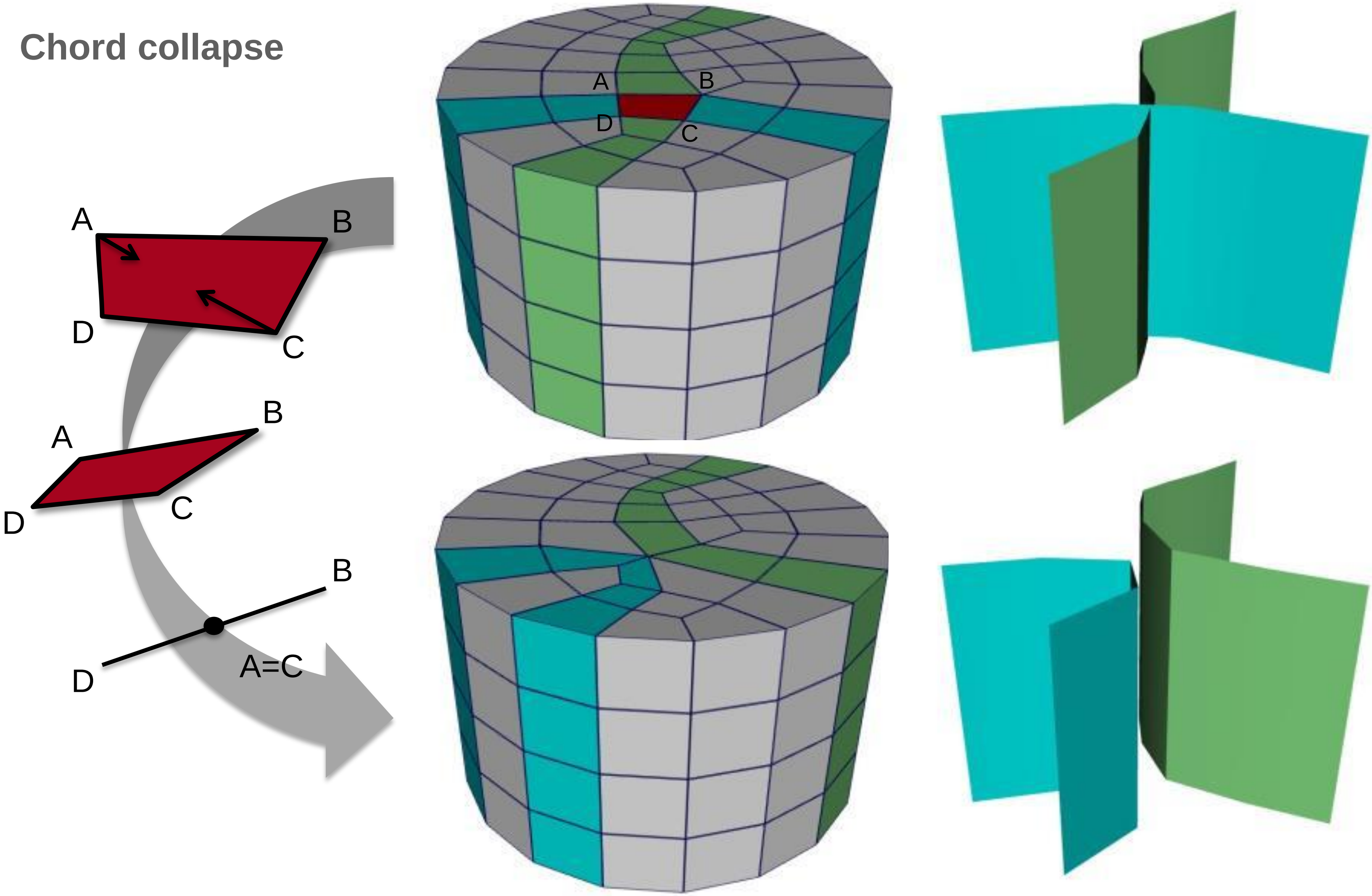
## Sheet extraction



M. J. Borde, S. E. Benzley, J. F. Shepherd, Hexahedral Sheet Extraction, proceedings of the 11th IMR, 2002

# Reliable operations to modify hexahedral meshes

## Chord collapse

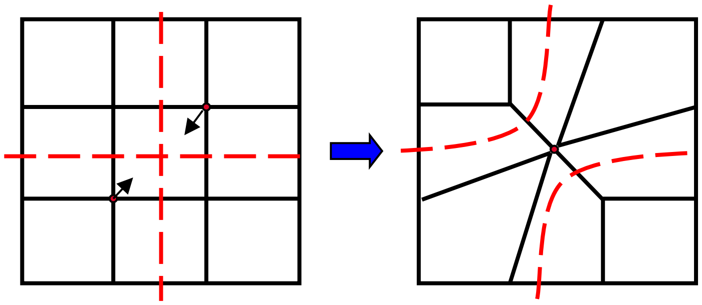


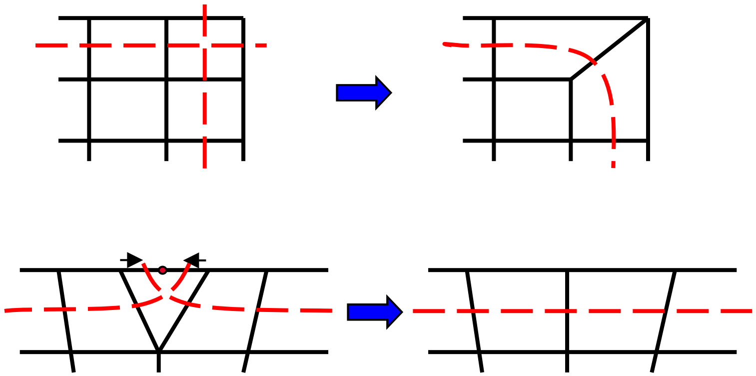

- Removes a complete column of hexahedral elements

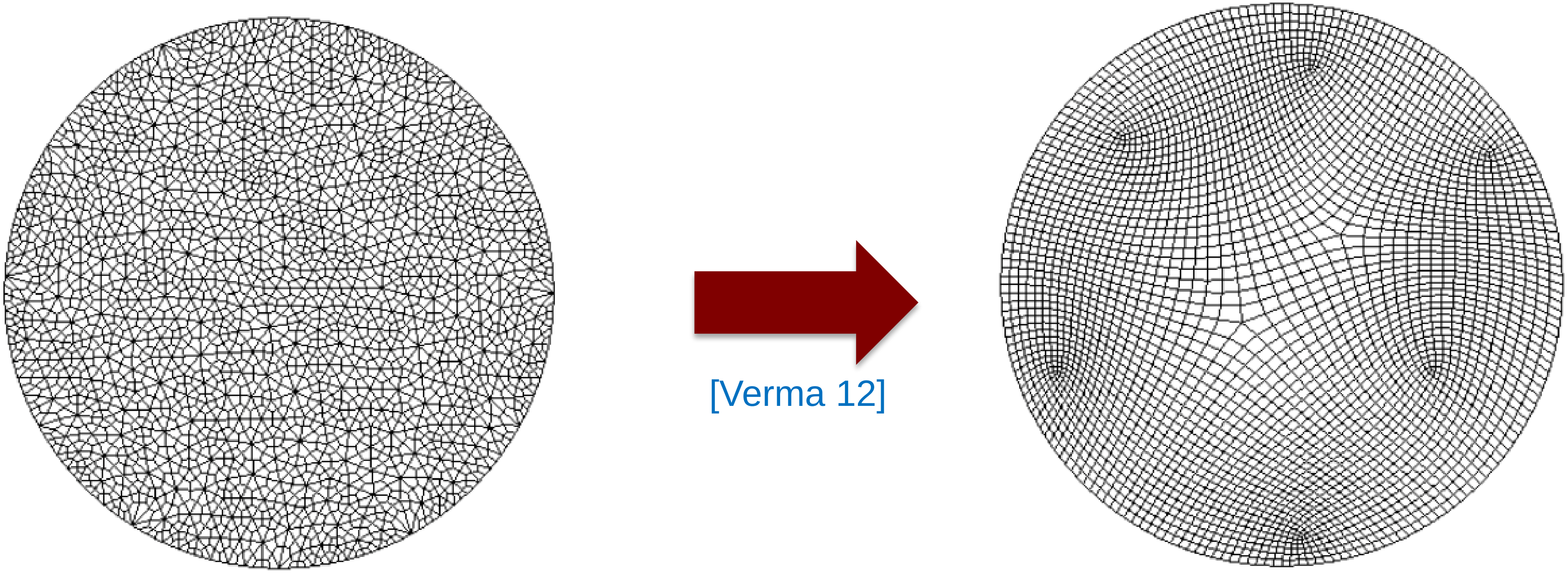
# RELIABLE OPERATIONS TO MODIFY QUADRILATERAL MESHES

## Local modification can be performed for 2D quadrilateral meshes

- Face collapse
- Boundary face collapse







[Verma 12]

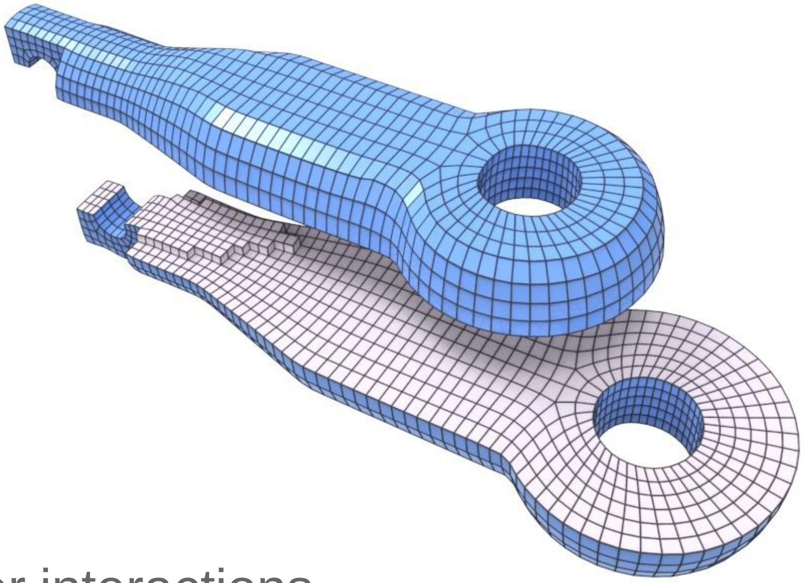
# OUTLINE

Why is it so difficult to generate full hexahedral meshes?

Which families of algorithms?

- Sweeping
- Geometric / Topological advancing-front approaches
- Medial axis
- Cartesian idealization –Submapping and polycubes
- Frame fields

# HOW TO COMPARE ALGORITHMS?



**Geometric quality**

1. Structure
2. Low distortion of the cells
3. Geometric boundary alignment
4. Size constraint

**Degree of automaticity**

Totally automatic OR requires some user interactions

**Genericity of the geometric domain**

Any type of objects or restricted to specific types of objects

**Respect a pre-meshed boundary**

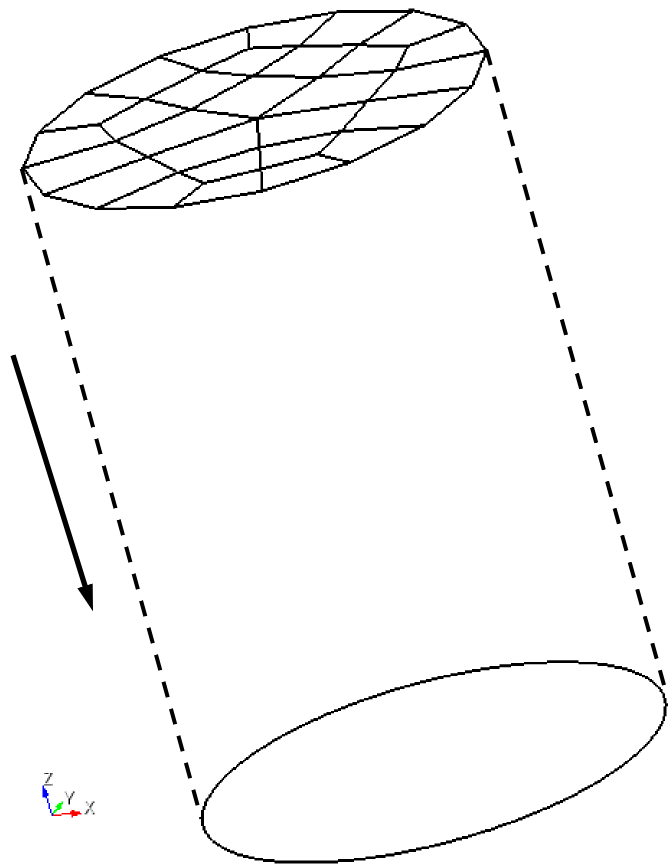
| | |
|---|---|
| Structured | ■ (green) |
| Boundary alignment | ■ (yellow) |
| Element size handling | … |
| Industrial maturity | … |
| Genericity | … |
| Respect of a boundary mesh | … |

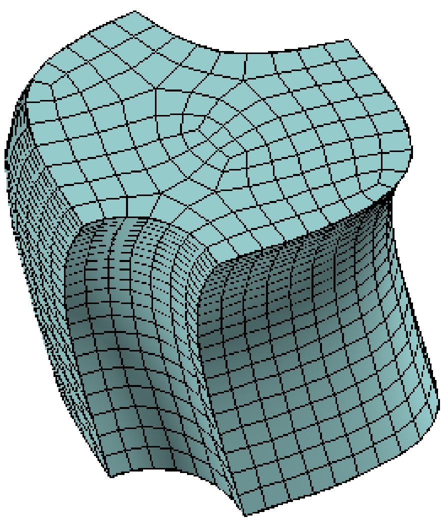
■ YES ■ NO ■ It depends

# SWEEPING 1-1

The mesh of the source surface is swept until reaching the target surface

Sweeping direction





[Blacker 97] [Roca and Sarrate 10]

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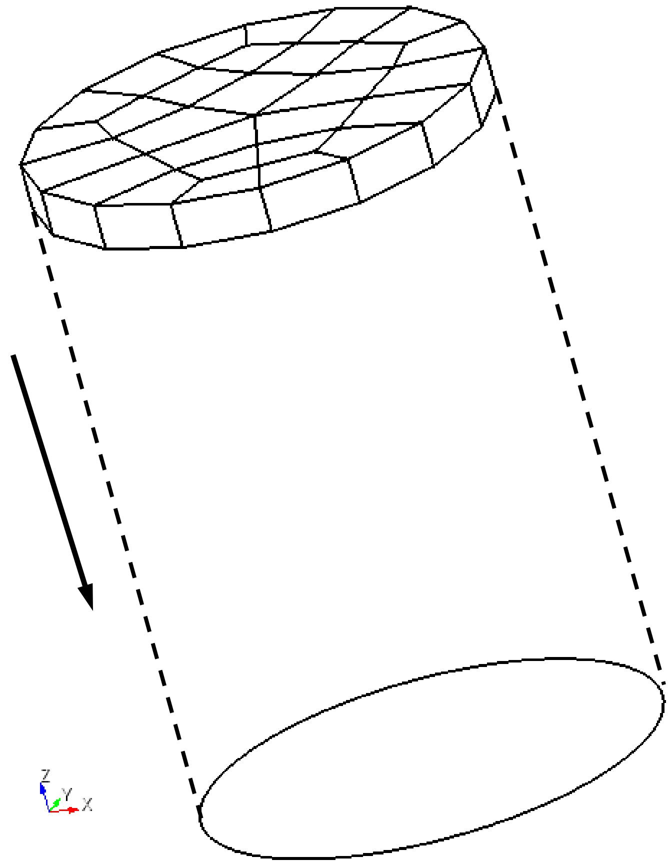


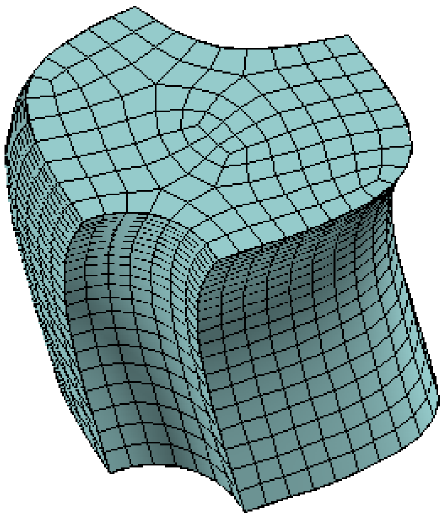

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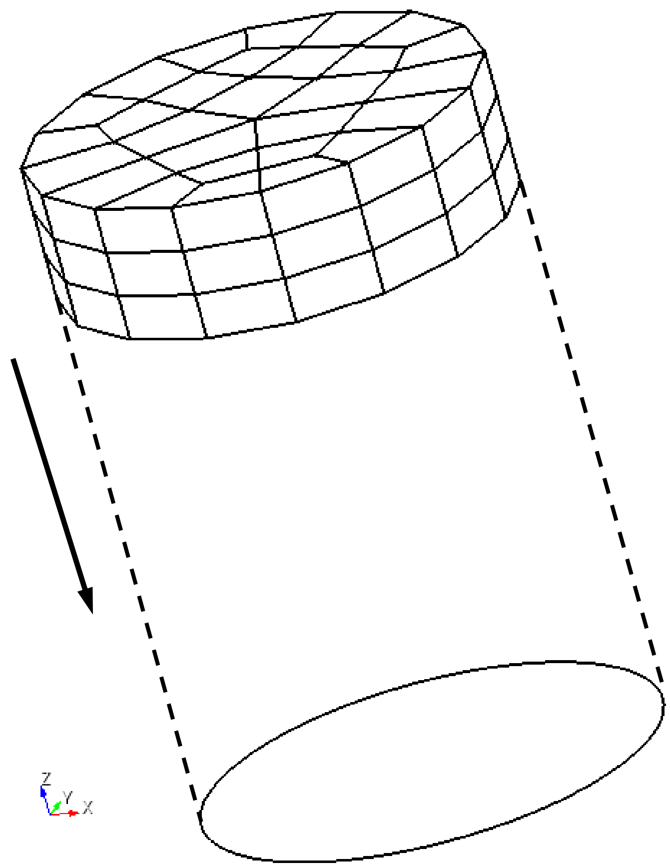

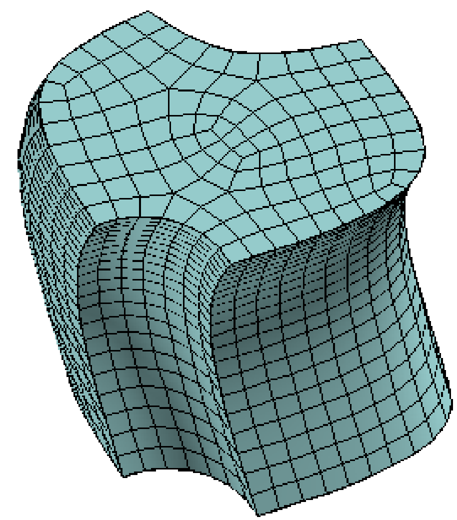

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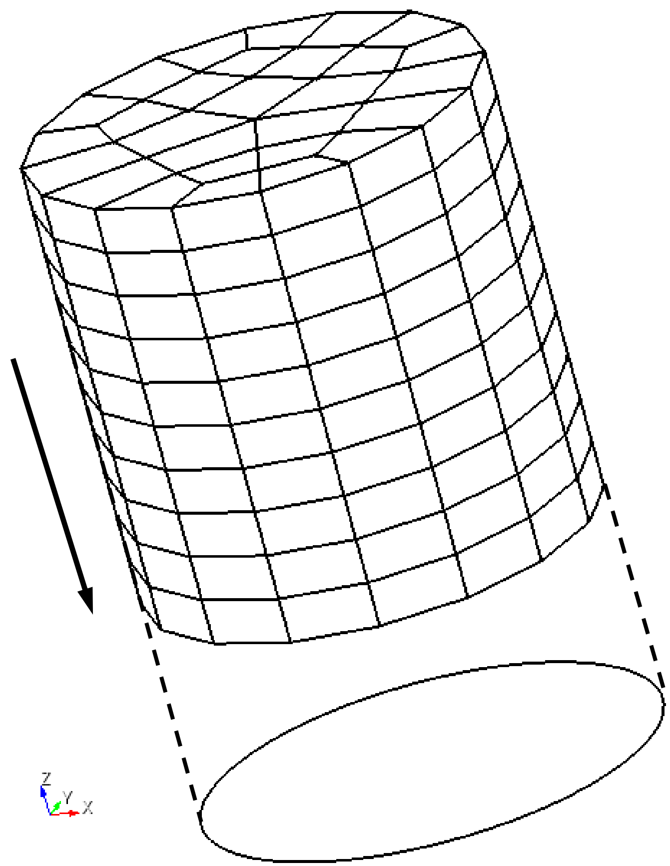


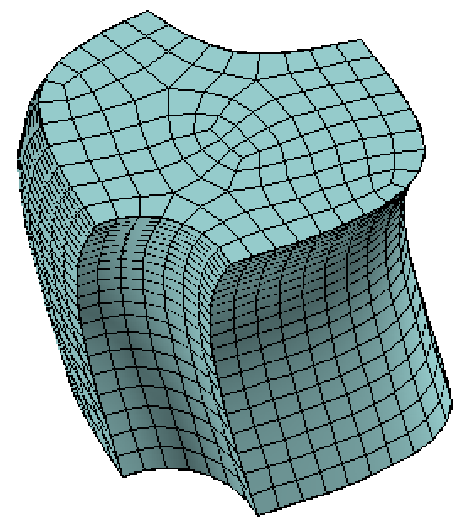

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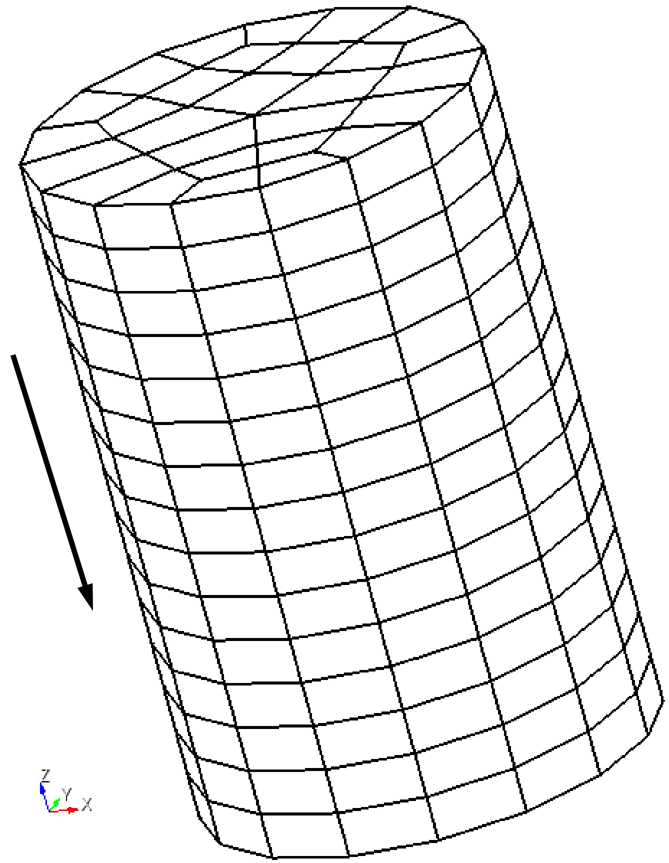

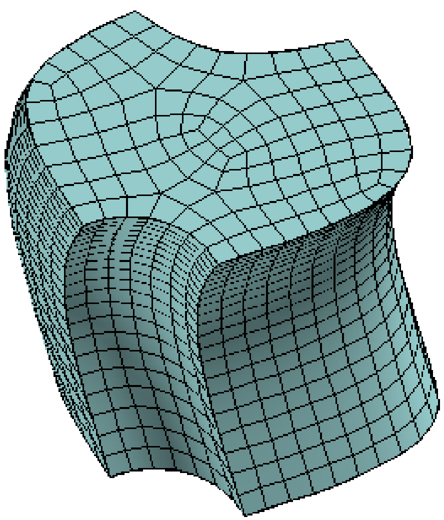

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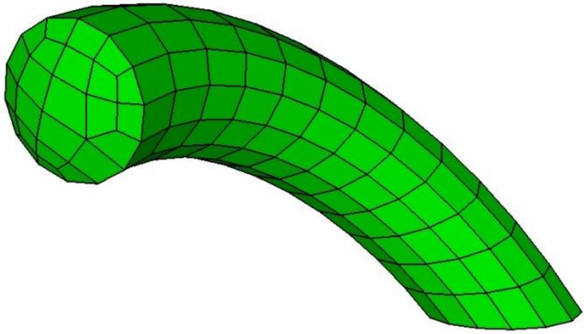
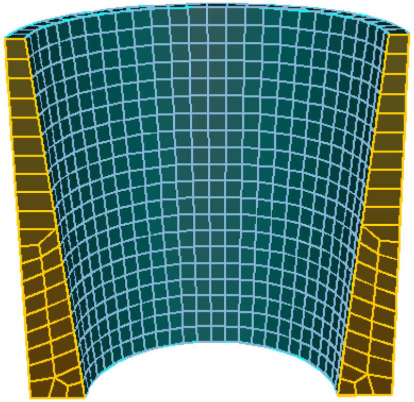
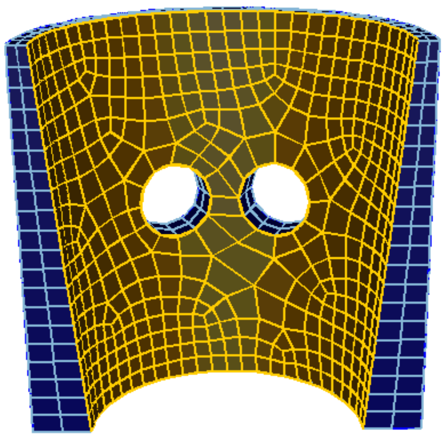
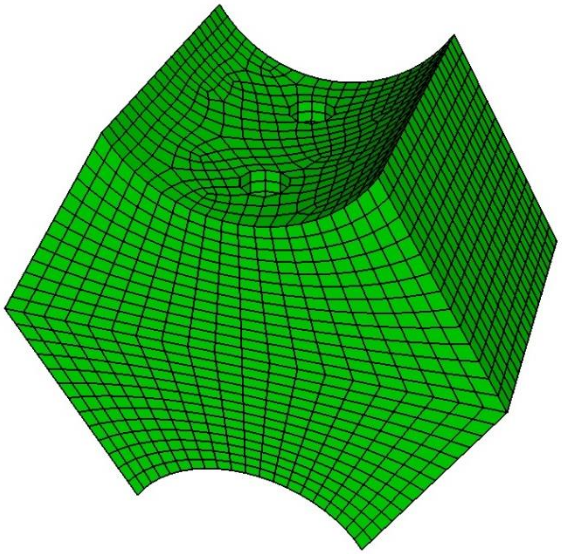
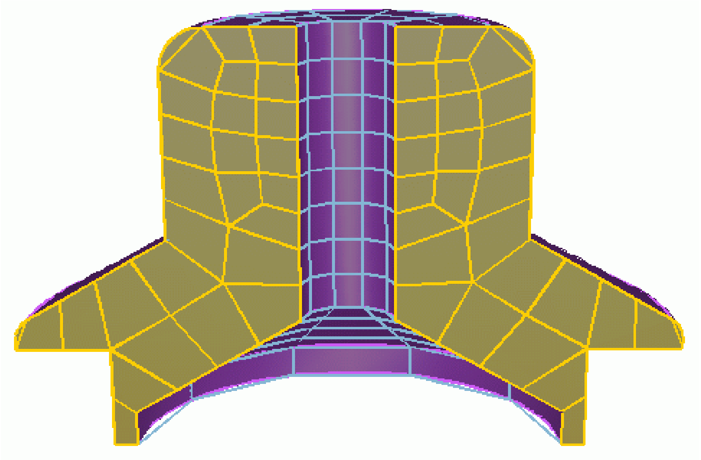


- Source and target surfaces can be non planar
- Shape and size variations are possible during the sweeping process
- Sweeping direction is not necessary linear





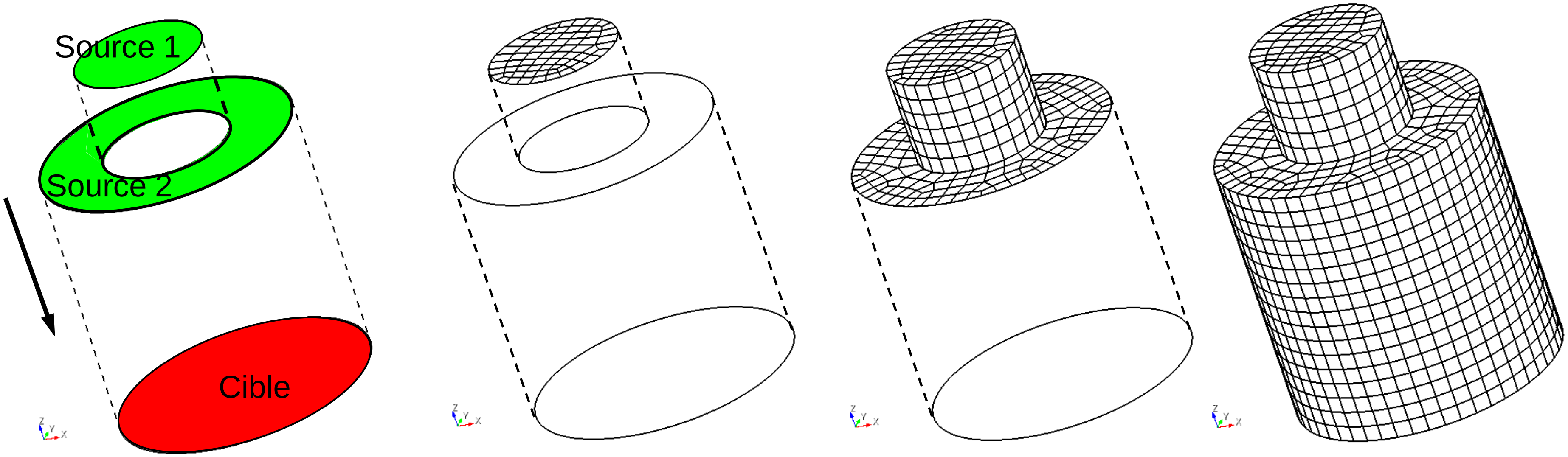




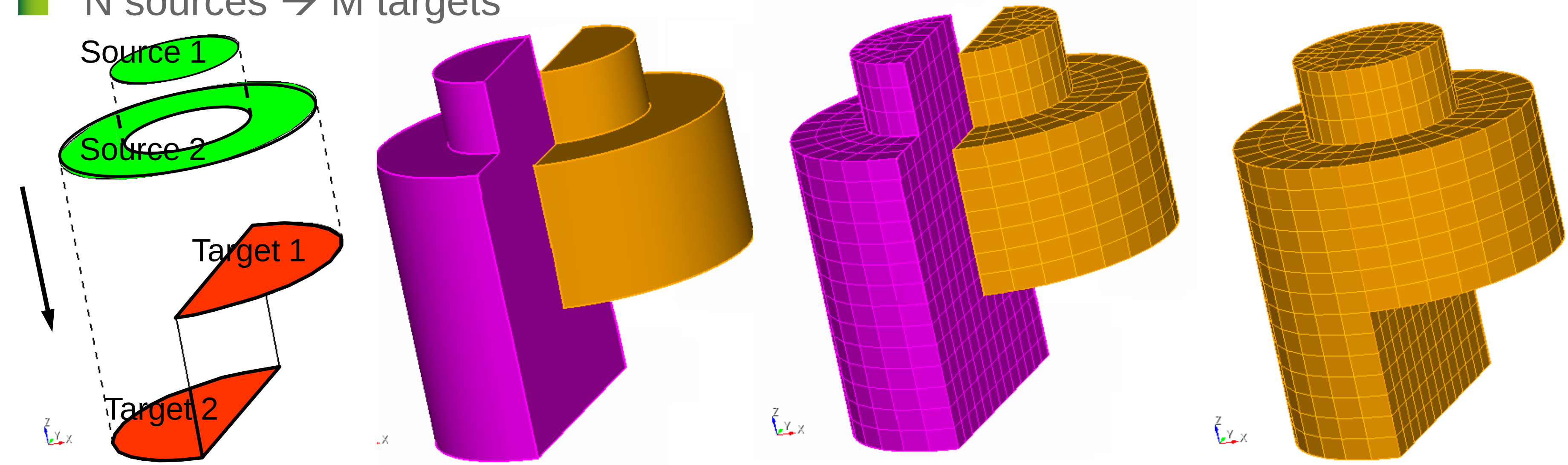
[Blacker 97] [Roca and Sarrate 10]

# SWEEPING N-1 AND N-M

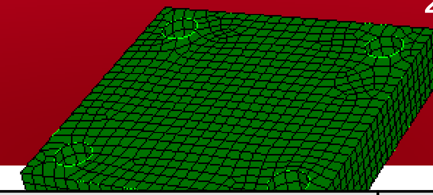
- N sources → 1 target




- N sources → M targets

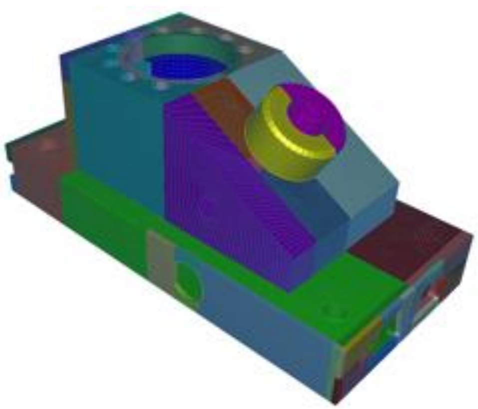


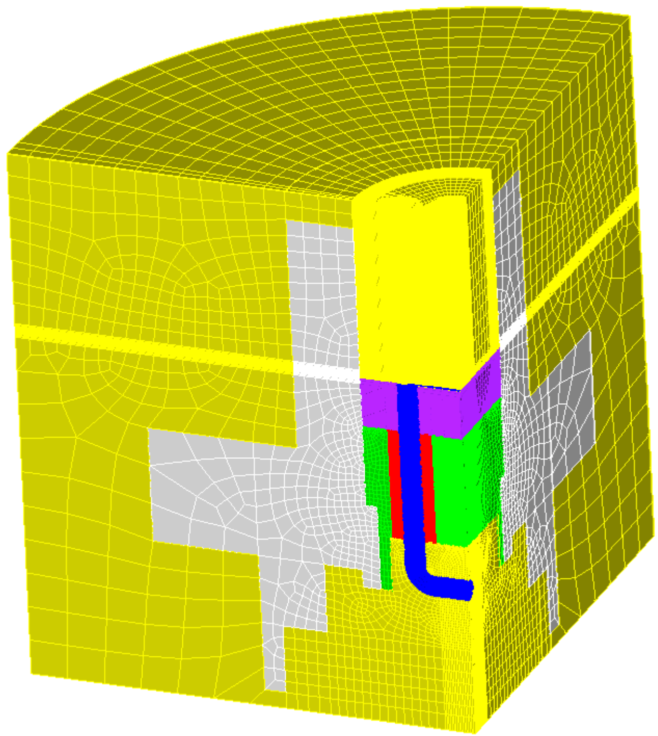
# SWEEPING N-M

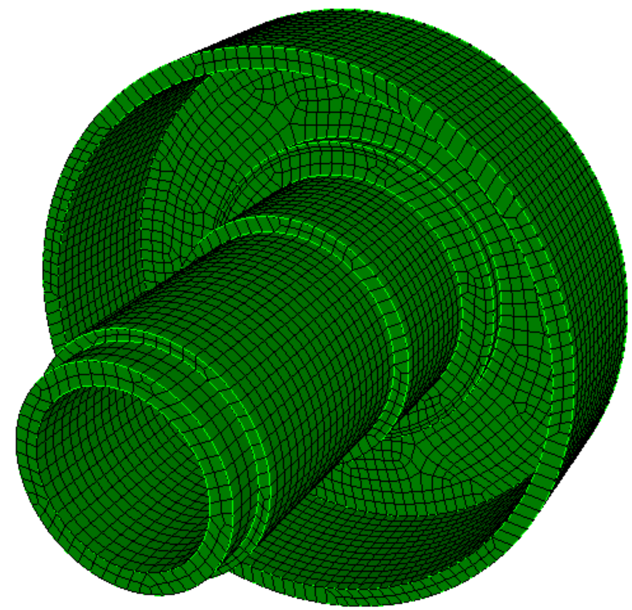


| | |
|---|---|
| Structured | 🟧 |
| Boundary alignment | 🟩 |
| Element size handling | 🟩 |
| Industrial maturity | 🟩 |
| Genericity | 🟥 |
| Respect of a boundary mesh | 🟥 |

- Geometric decomposition into meshable blocks *(hand-made most of the time)*
- Each block is meshed with taking care of conformity constraints





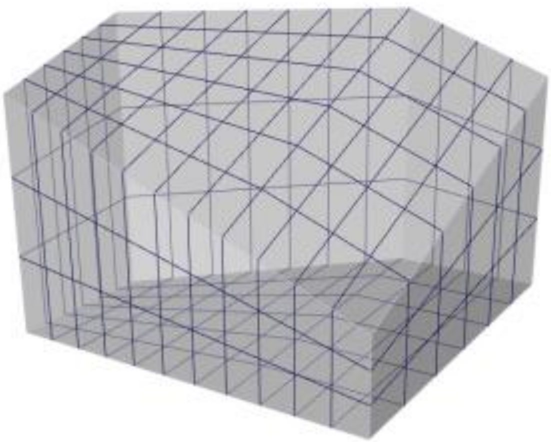


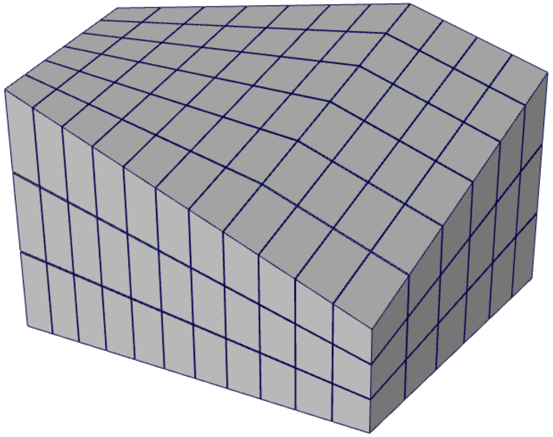
[Blacker 97] [Roca and Sarrate 10]

cubit.sandia.gov

# GENERATION OF A HEX. MESH FROM A QUAD MESH

## The most constrained problem is mostly solved by advancing-front algorithms





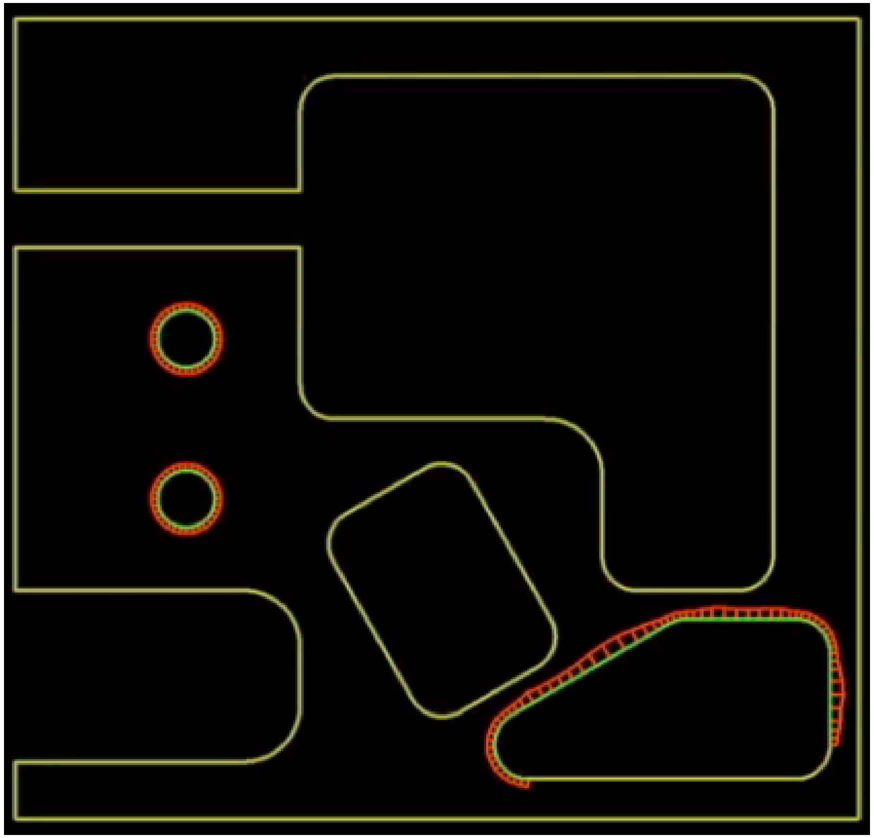
### Geometric approaches

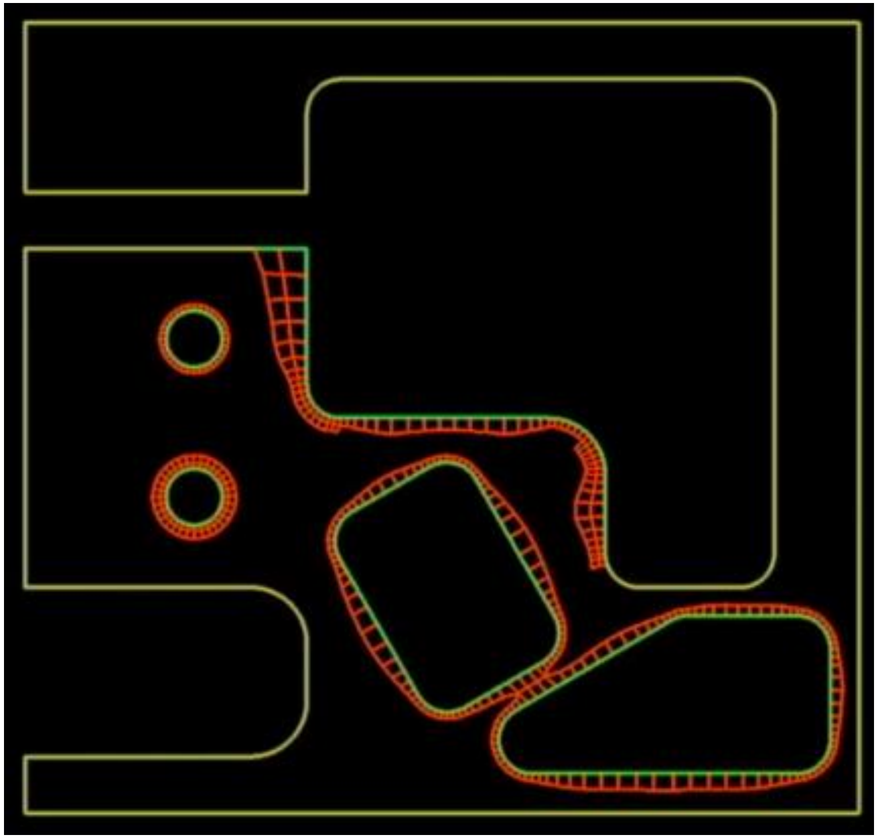
- Plastering [T. BLACKER 93]
  - Hexahedral elements added one per one
- H-Morph [S. OWEN 00]
  - As plastering but uses a tet. Mesh to solve geometric queries

→ **All of them fail in the termination process**

**BUT recently, global approaches using frame fields** [HUANG ET AL. 11] [LI ET AL 12]

# Geometric advancing front – Main principle in 2D (1/3)

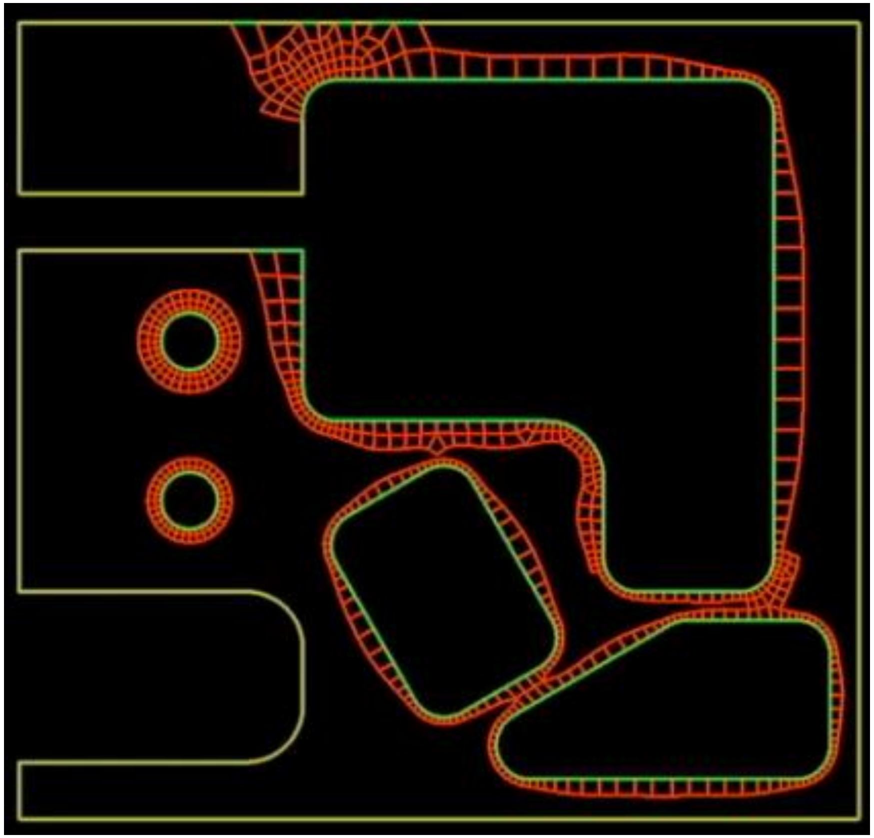


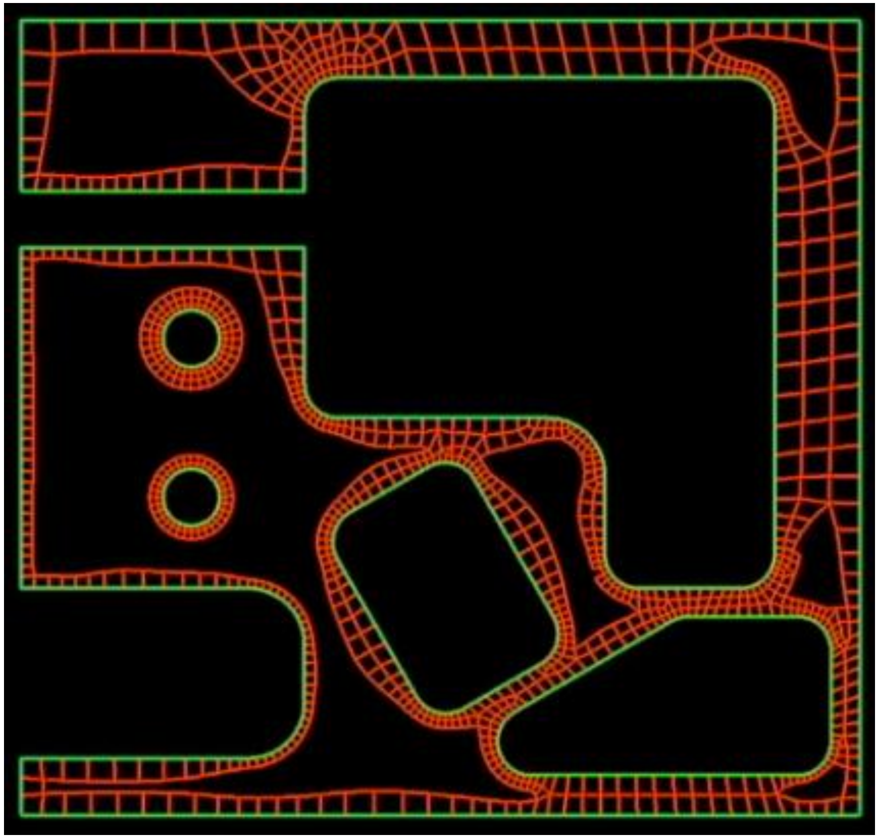


*Advancing-front mesh generation*

- Local decisions based on the elements' shape

# Geometric advancing front – Main principle in 2D (2/3)

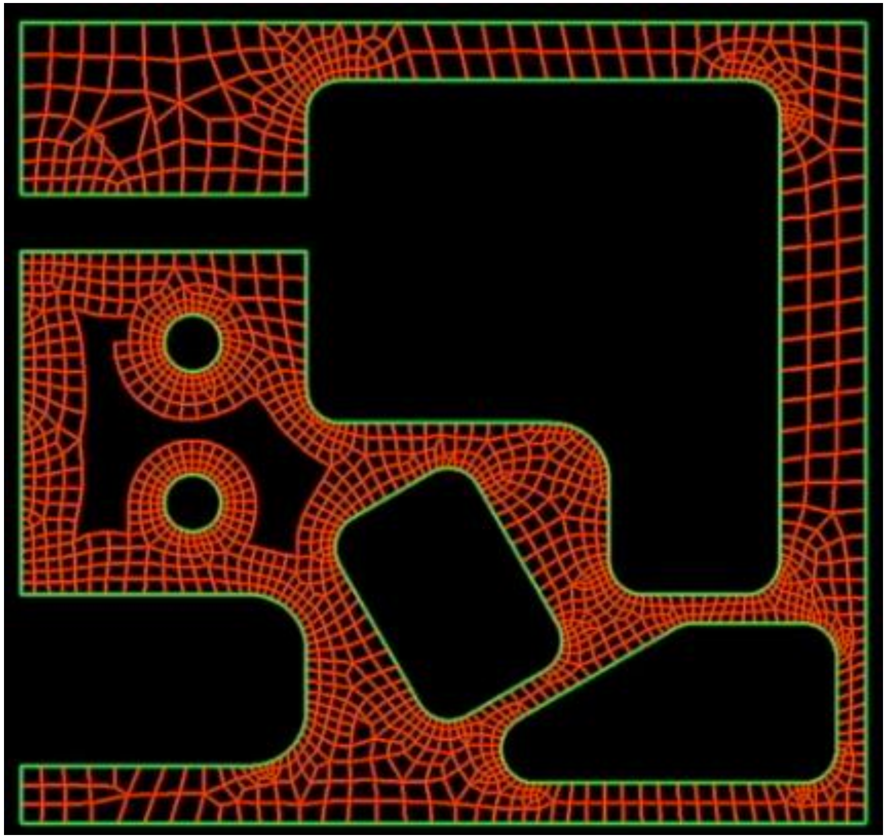


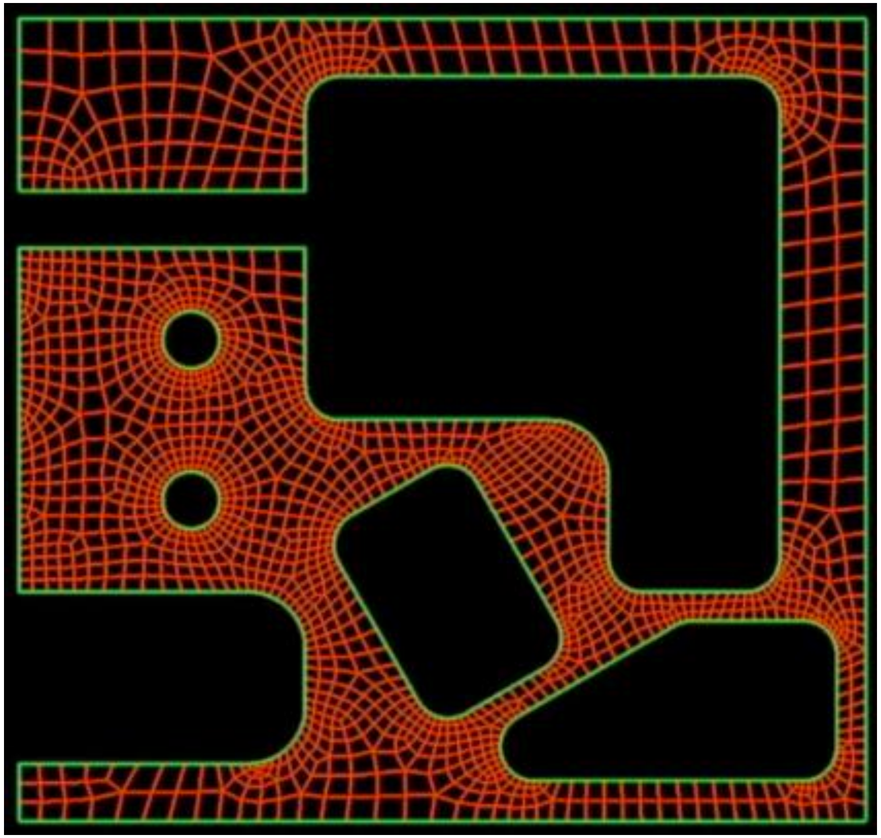


*Advancing-front mesh generation*

- Local decisions based on the elements' shape

# Geometric advancing front – Main principle in 2D (3/3)





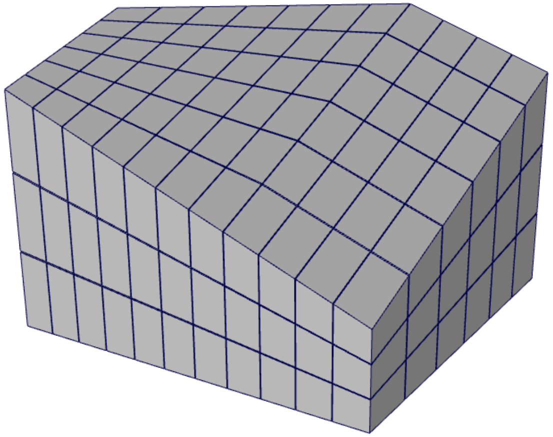
*Advancing-front mesh generation*

- Local decisions based on the elements' shape

# THE TOPOLOGICAL PROBLEM

Topological approaches
topology is solved first, restrictions about the surface mesh

- Whisker weaving [Tautges et al. 96, N. Folwell and S. Mitchell 98]
  - Local geometric conditions must be satisfied along the domain boundary
- Recursive Bisection [Calvo and Idelsohn 00]
  - The domain is recursively split
- Dual Cycle Elimination and Shelling [Muller-Hannemann 02]
  - Particular process for parallel loops and the sheet selection depends on geometry → extended in [M. Kremer and al.13]



# THE TOPOLOGICAL PROBLEM, FROM THE THEORY POINT OF VIEW

Let Q be a topological quadrilateral mesh of a connected surface in $R^3$ such that :

- Q has an even number of quadrilaterals and no odd cycle in Q bounds a surface inside the interior domain. (True if genus-zero).

Then Q can be extended to a topological hexahedra mesh of the interior domain [Erickson 14]

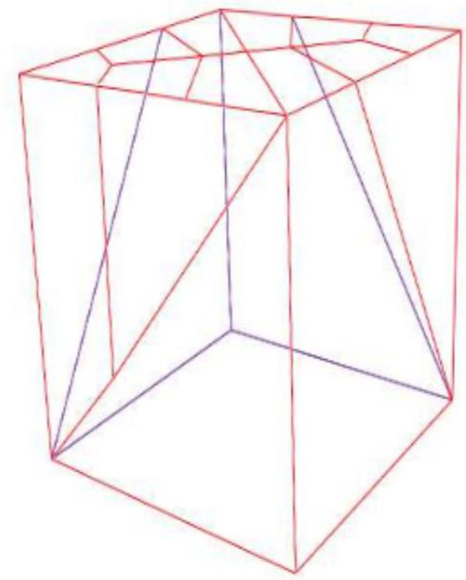
There is a constructive proof… It only requires to find a solution of 20 or 22 quadrilaterals *buffer cubes*. As stated the existence of such a hex mesh is guaranteed by Thurston and Mitchell's proof, *it is not difficult to construct explicit hex meshes for these subdivided cubes by hand*…

# THE TOPOLOGICAL PROBLEM, FROM THE THEORY POINT OF VIEW

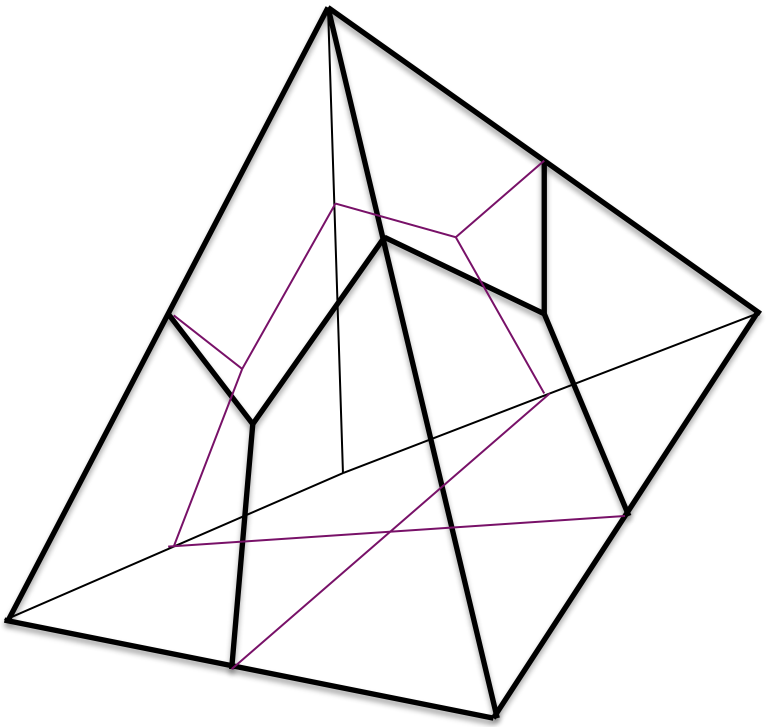
Up to now, no solution has been found by hand !

Using [Carbonera and Shepherd 06], a solution has been found with **76881** hexes.

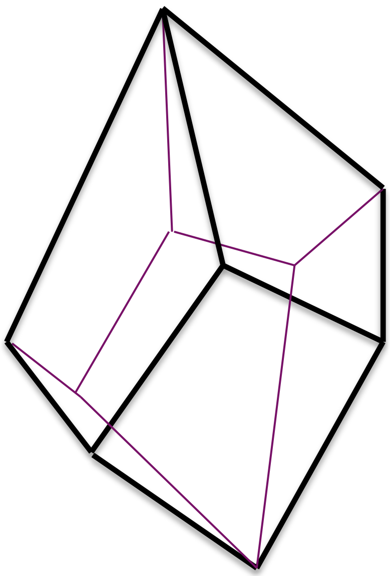
It can be shown that it needs at least 12 hexes [Weill 16]



# CURRENT BEST ‘KNOWN’ SOLUTIONS



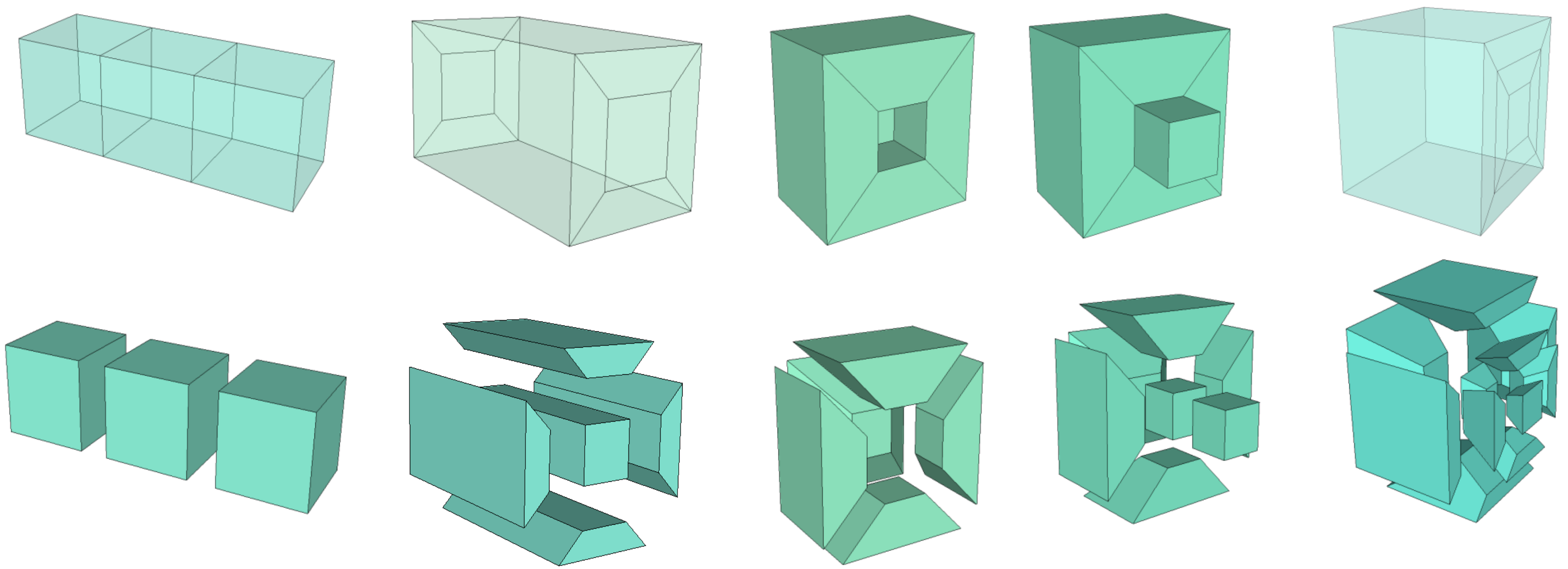
88 hexes [Yamakawa, Shimada 10]



38440 hexes [Carbonera and Sheperd 10]

# FROM A TOPOLOGICAL TO GEOMETRICAL MESH

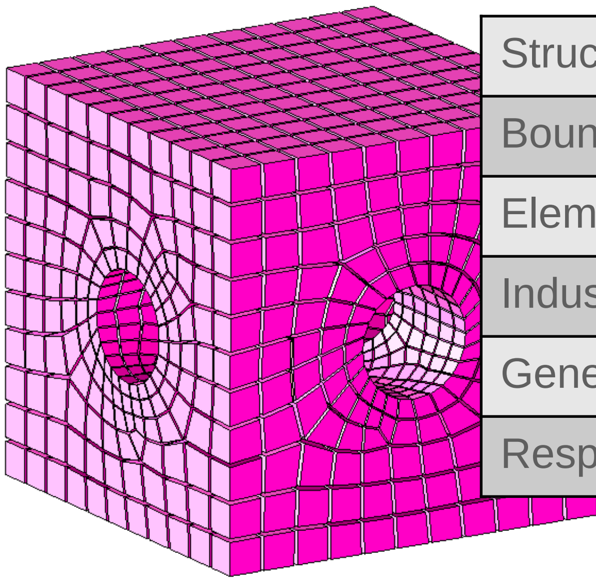
Same topological surfaces !



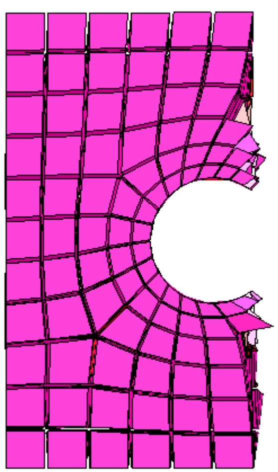
Up to now, no test to tell if a topological mesh can lead to a geometric mesh

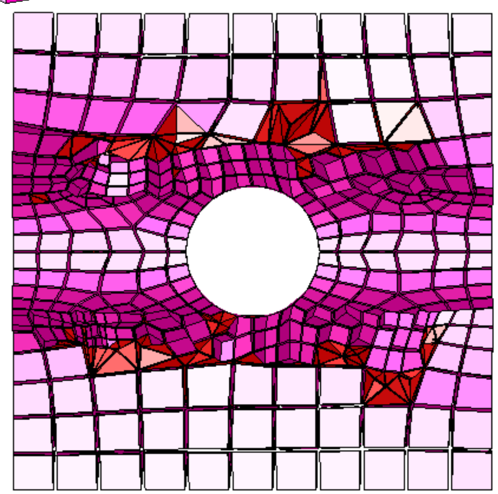
# ADVANCING-FRONT LIMITATIONS IN 3D

## Geometric advancing-front



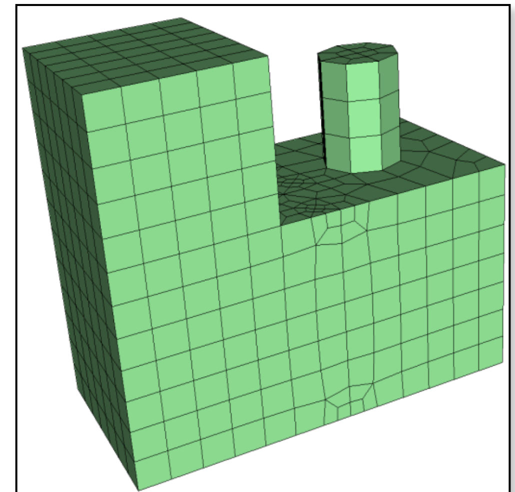
| | |
|---|---|
| Structured | 🟥 |
| Boundary alignment | 🟩 |
| Element size handling | 🟥 |
| Industrial maturity | 🟥 |
| Genericity | 🟩 |
| Respect of a boundary mesh | 🟩 |

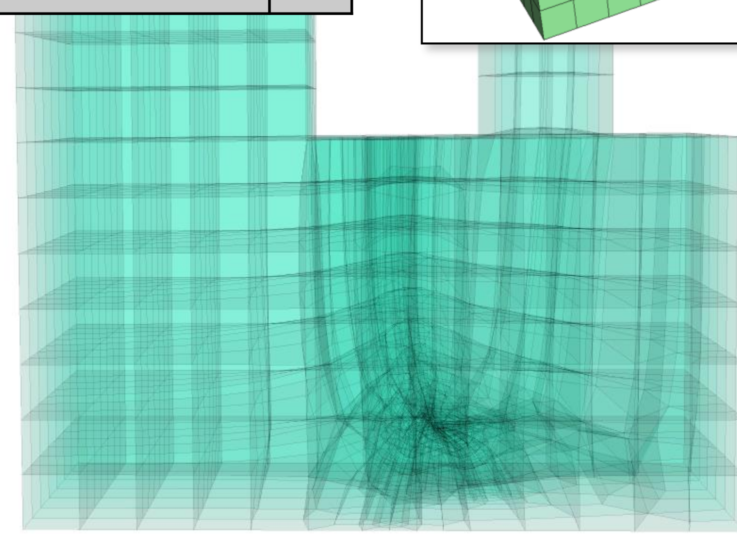




[S. Owen 00]

## Topologic advancing-front



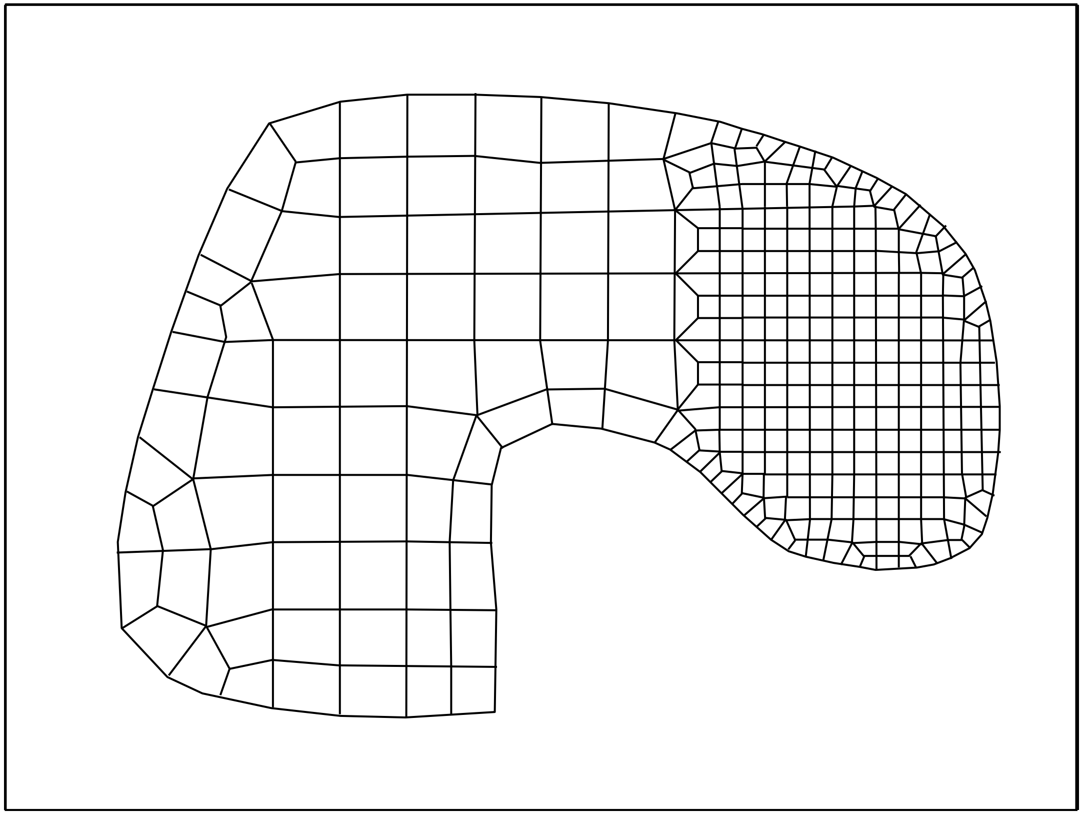


[Ledoux and Weill 08]

# OVERLAY-GRID METHODS

Existing automatic and robust solution

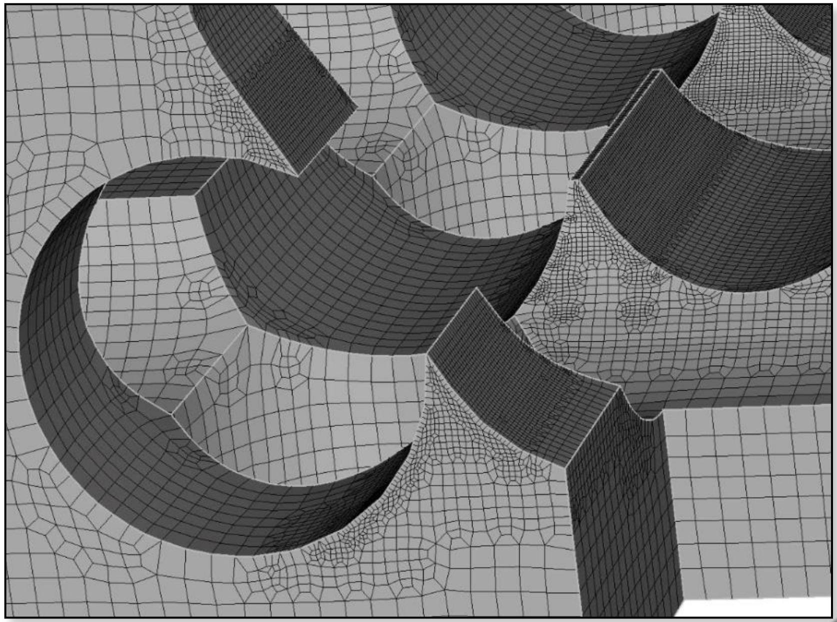
Very used for biomedical applications and any applicative field that works with free-form surfaces or iso-surfaces without any sharp edges

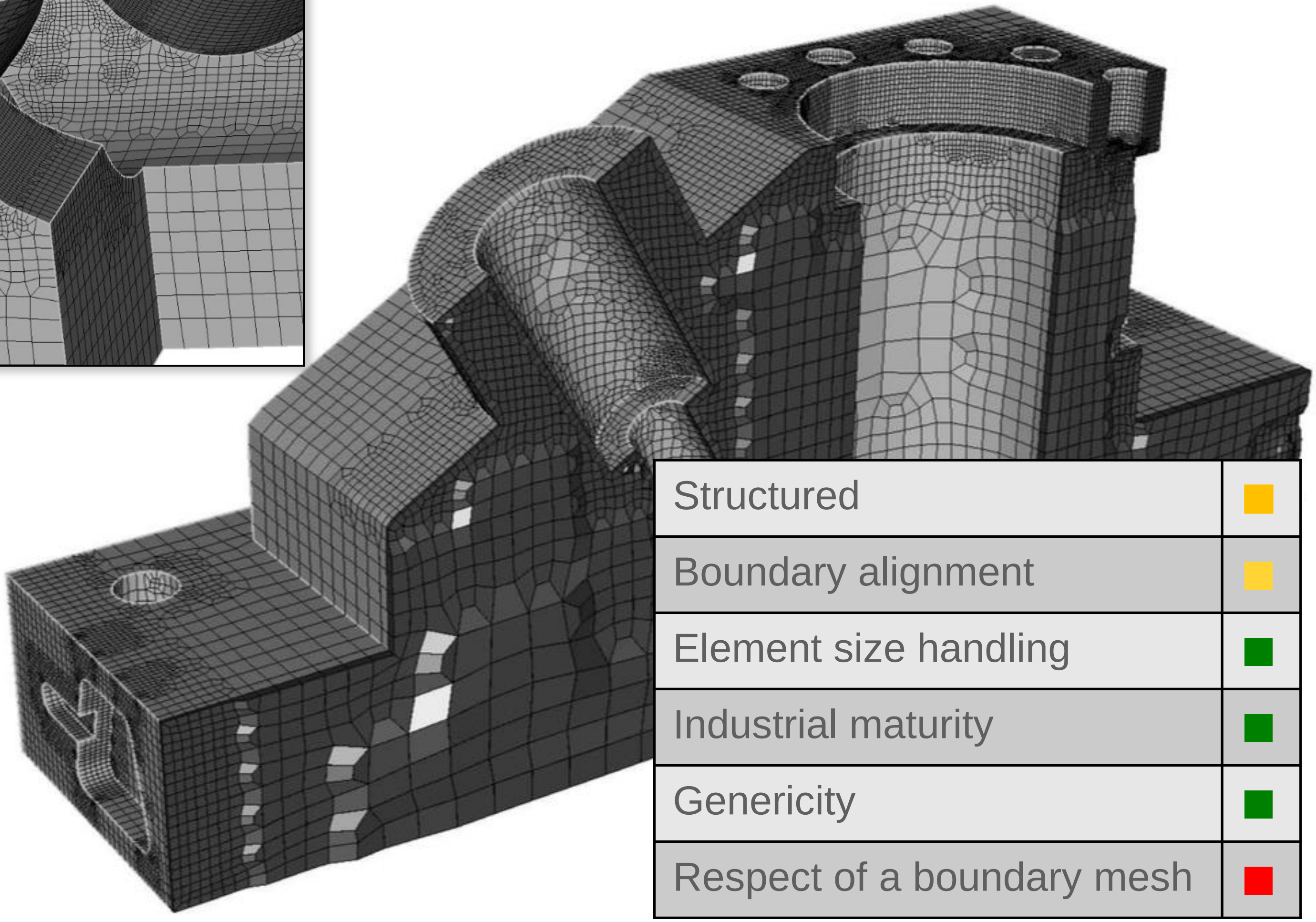


Disadvantages

- Grid orientation sensitive
- Worst quality elements are along the boundary
- Boundary topology can be lost if the grid discretization is not well-adapted

# OVERLAY-GRID BASED METHODS – 3D CAD





| | |
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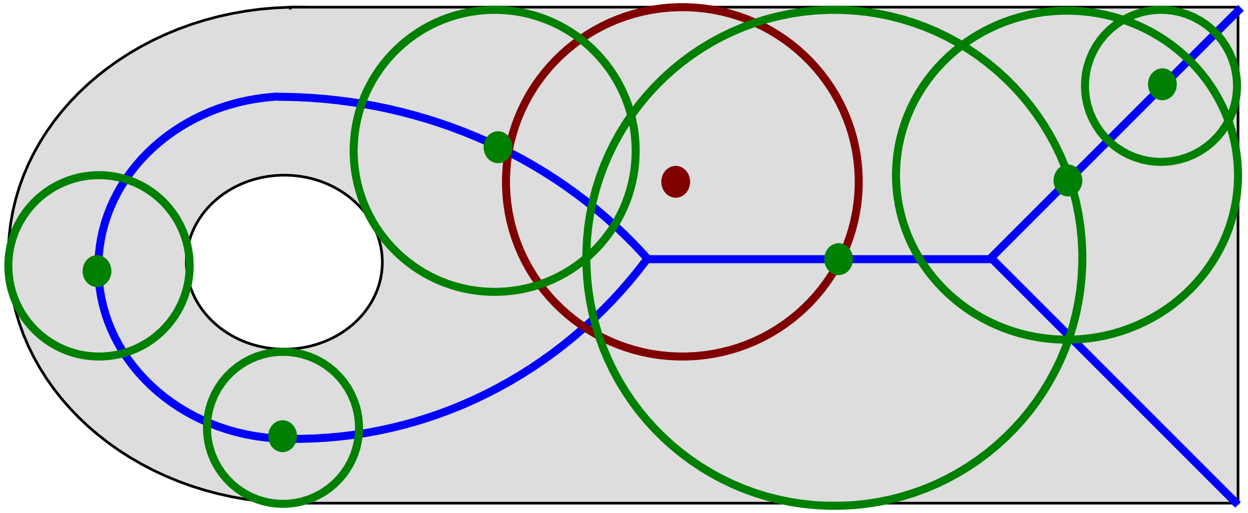
[Maréchal 09]

# USING THE MEDIAL AXIS – PRINCIPLE

- The medial axis is a skeletal representation of a geometric object

Let Ω be a geometric domain, the medial axis MA(Ω) of Ω is defined by the set of points **p** in Ω such that **U(p)** touches the boundary of Ω more than once, with **U(p)** the largest circle centered in **p** that is entirely within Ω.

Ω



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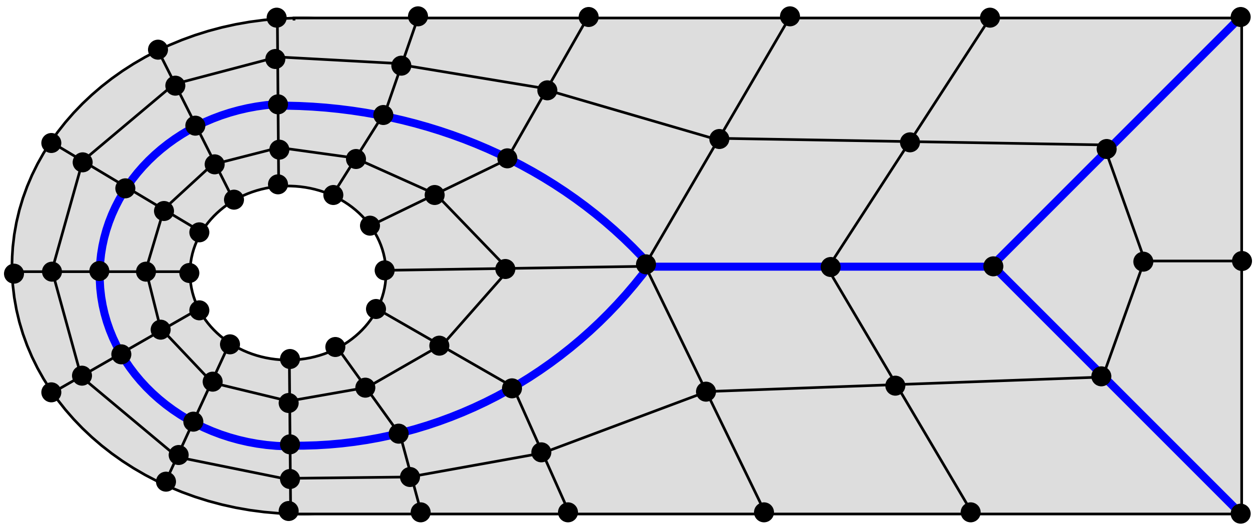
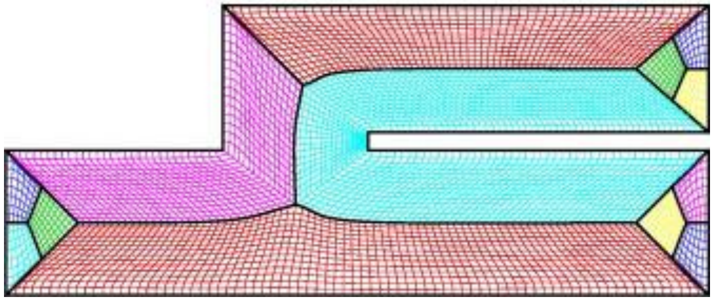
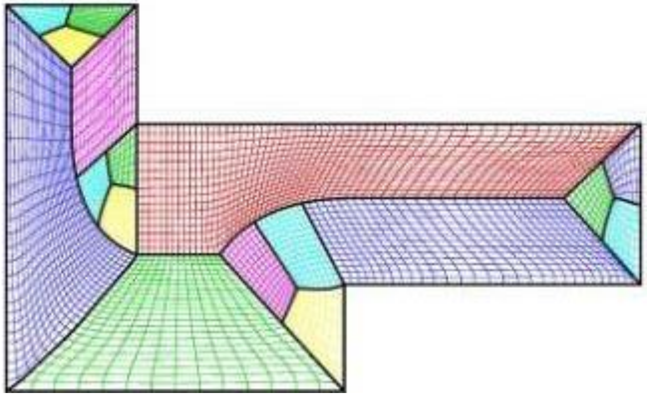


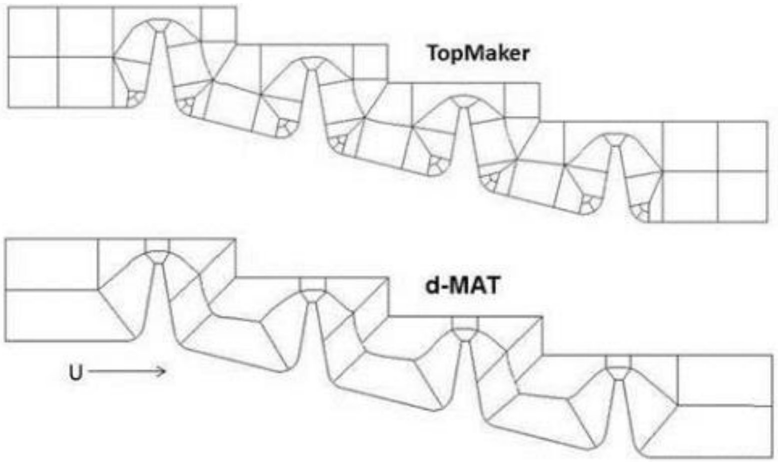
# USING THE MEDIAL AXIS – 2D EXAMPLES

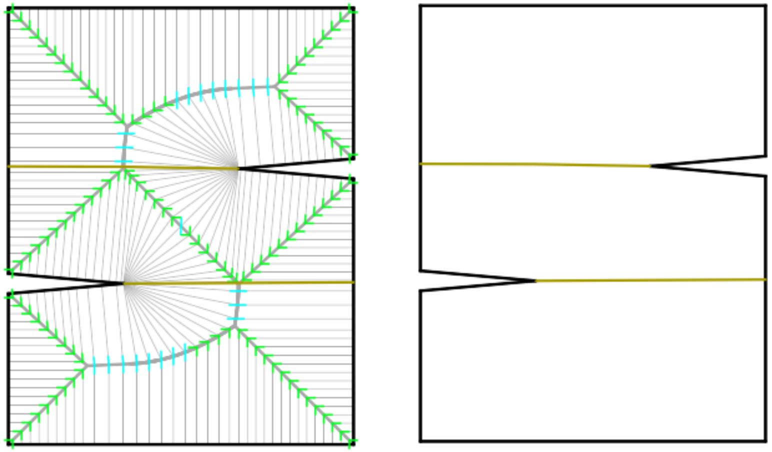
- Straightforward block meshing [Hao et al. 11]
  (-) Sharp corners are badly captured





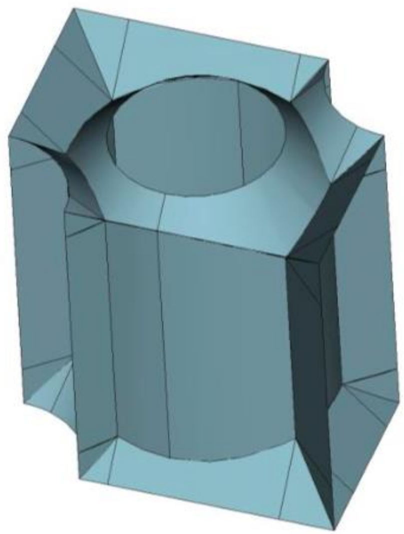
- With post-processing [IMR13] [Fogg et al. 14]
  (-) Mesh singularity often remains on the medial axis (not always what users expect)

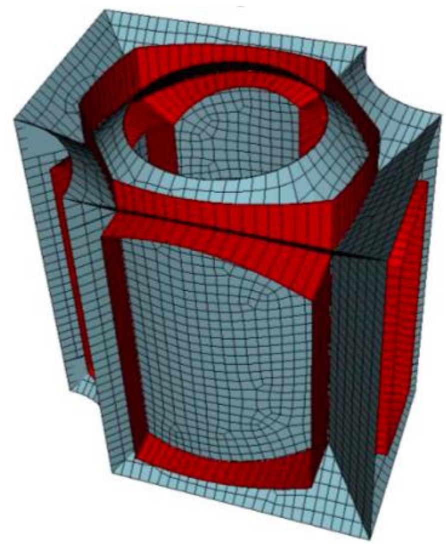


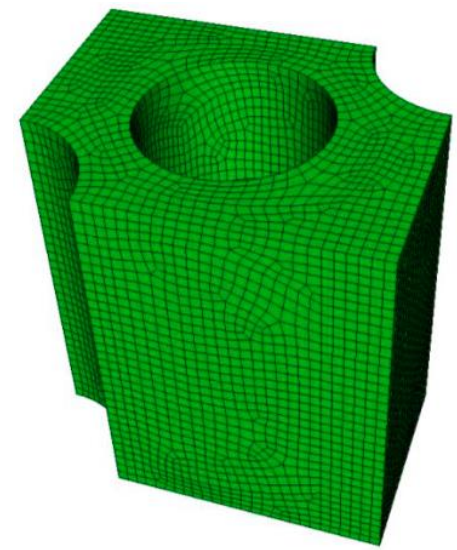



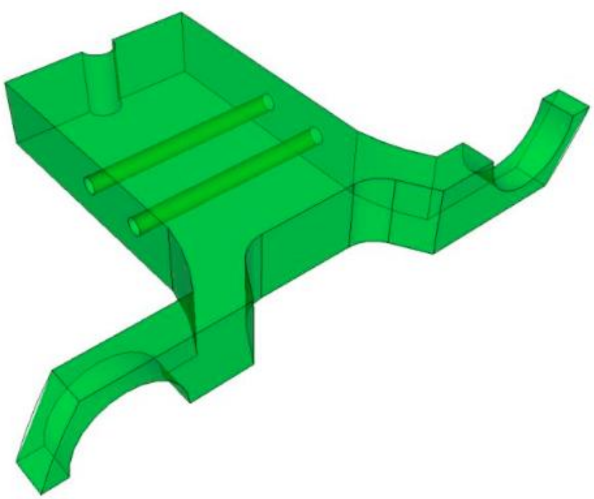
# USING THE MEDIAL AXIS – 3D EXAMPLES

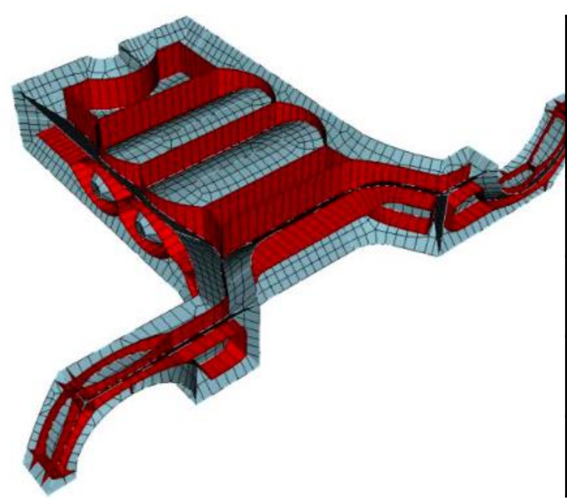
Meshing of the medial axis, then 1-1 sweeping in restricted areas [Quadros 14]











| | |
|---|---|
| Structured | |
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| Element size handling | |
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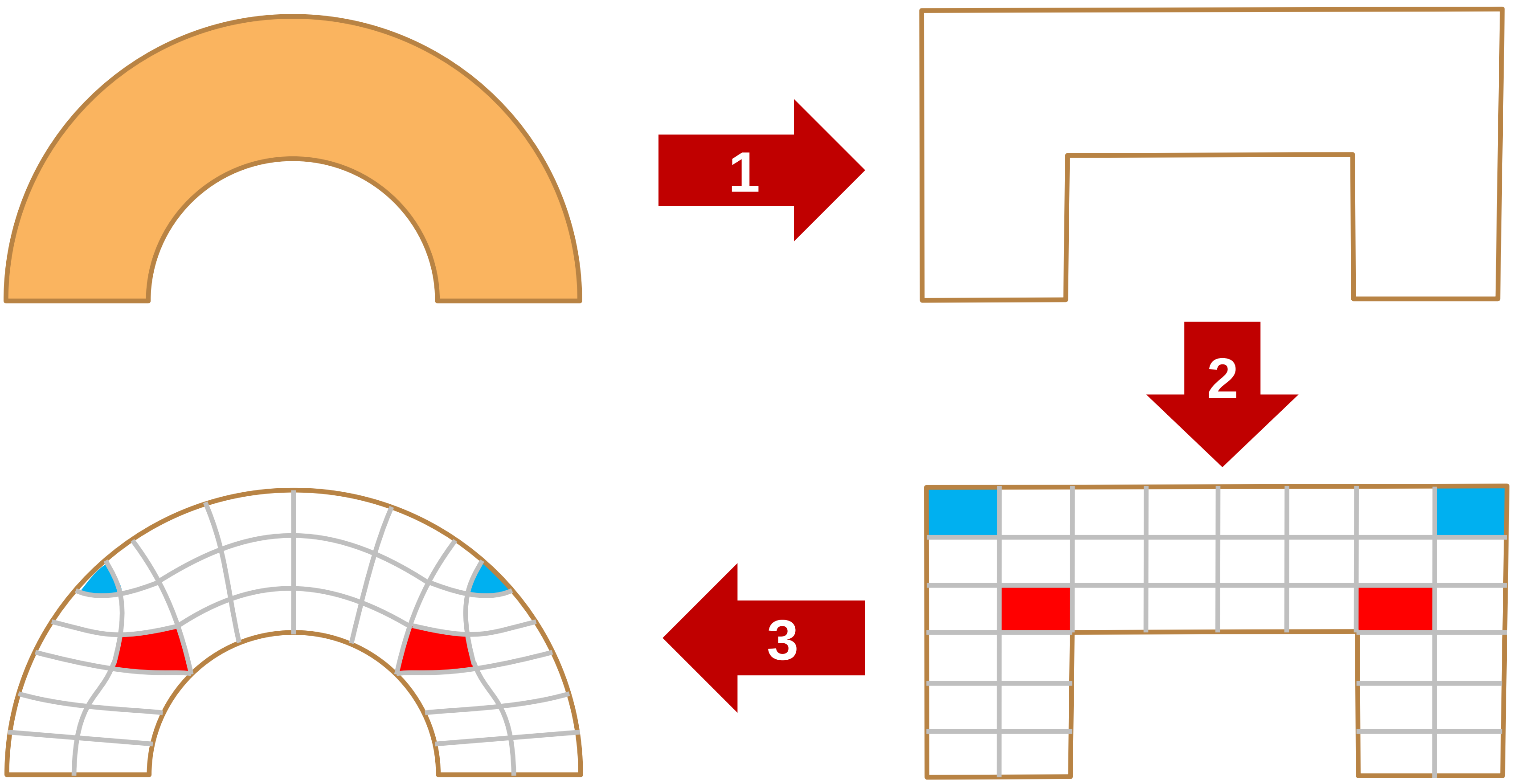
# CARTESIAN IDEALIZATION – MAIN PRINCIPLE

- Step 1 – **Convert** the geometric domain Ω into a polycube $\mathbf{P}_\Omega$
- Step 2 – **Mesh** the polycube $\mathbf{P}_\Omega$
- Step 3 – **Project** the mesh of $\mathbf{P}_\Omega$ onto Ω



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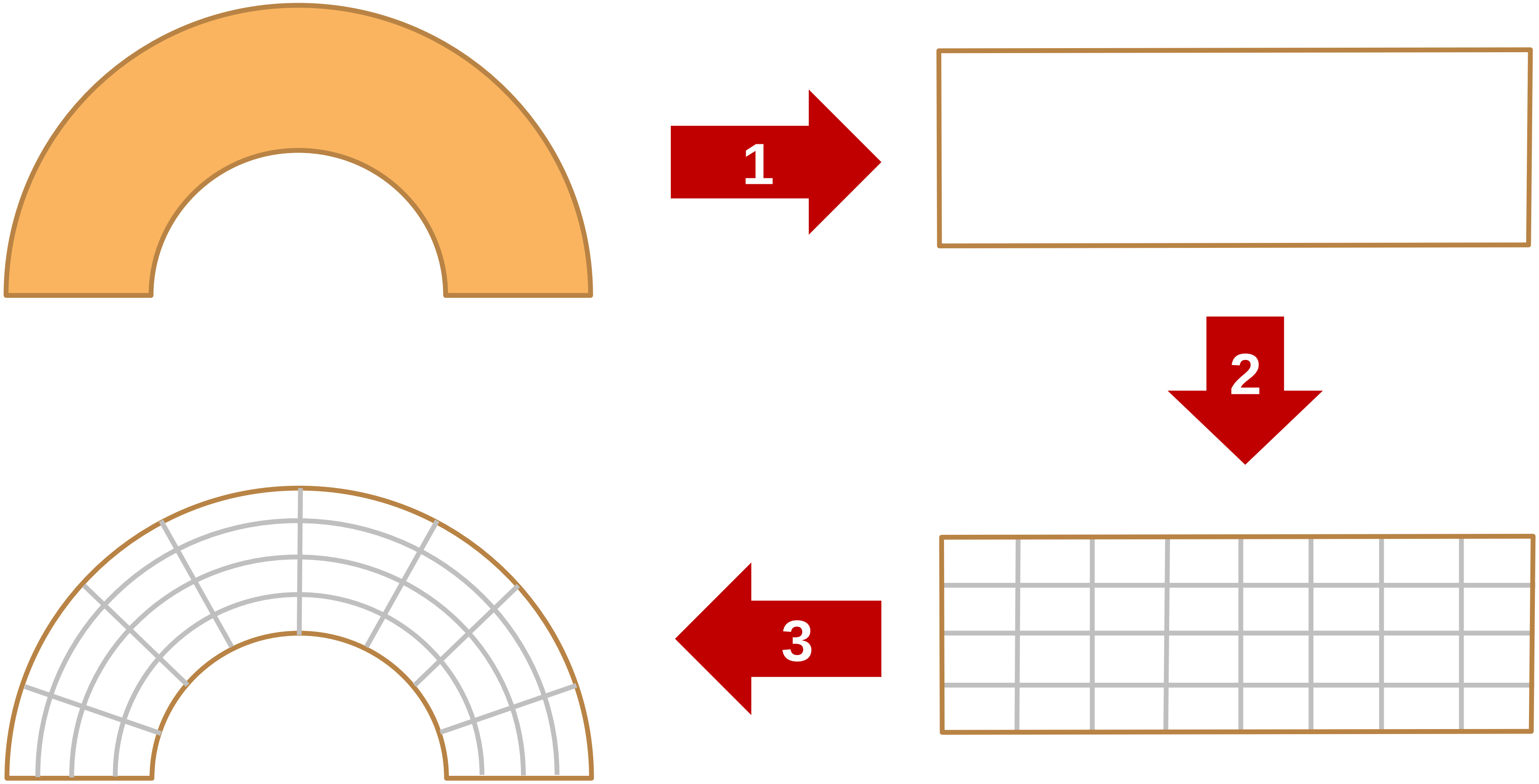


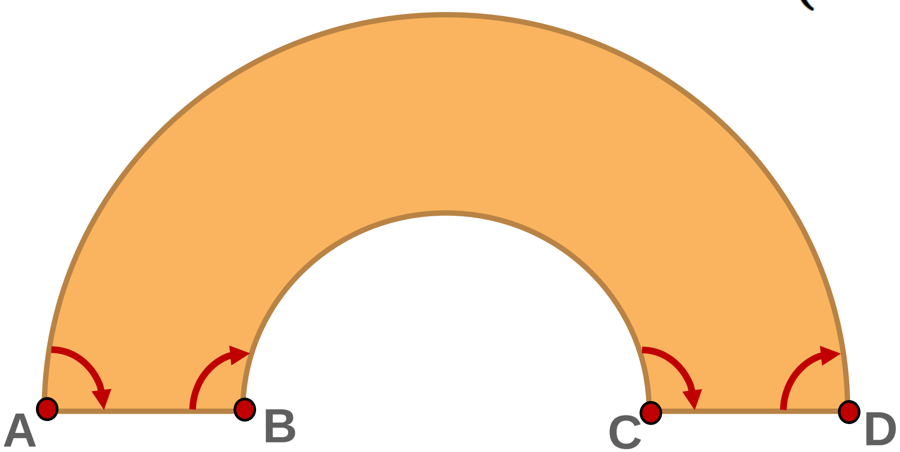
# CARTESIAN IDEALIZATION – MAIN PRINCIPLE

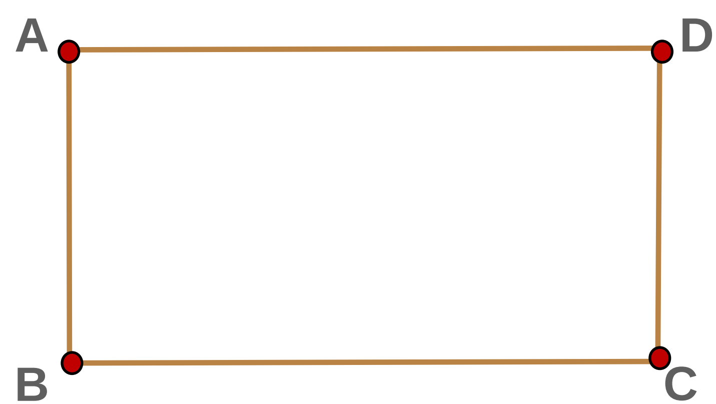
- Submapping approaches

  Solve a global boundary constraint problem [Ruiz-Girones et al. 10]


**Step 1** – Angle-based idealization

$$\begin{cases} \textit{side} & \text{the angle is } 0, \\ \textit{end} & \text{the angle is } \pi/2, \\ \textit{reversal} & \text{the angle is } -\pi, \\ \textit{corner} & \text{the angle is } -\pi/2. \end{cases}$$



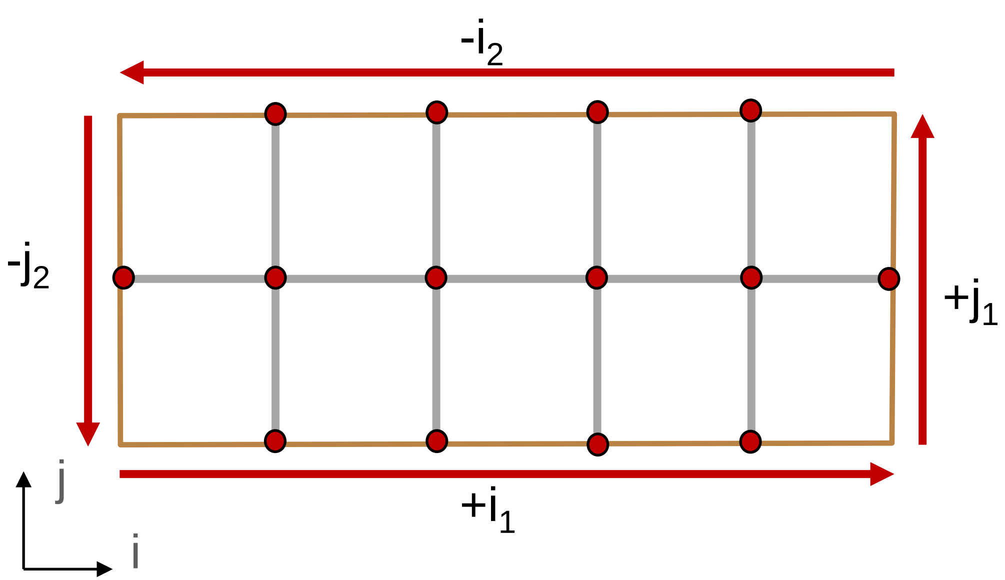
# CARTESIAN IDEALIZATION – MAIN PRINCIPLE

- Submapping approaches

Solve a global boundary constraint problem [Ruiz-Girones et al. 10]


**Step 3 –** Automatically decomposes surface into mappable regions based on assigned intervals + transfinite interpolation

$$\begin{cases} \min \sum_e \omega_e n_e + M, & \\ \text{constrained to:} & \\ \sum_{e \in I^+} n_e = \sum_{e \in I^-} n_e, & \\ \sum_{e \in J^+} n_e = \sum_{e \in J^-} n_e, & \\ M \geq \dfrac{n_e}{N_e}, & \text{for every edge } e \\ n_e \geq N_e, & \text{for every edge } e \end{cases}$$



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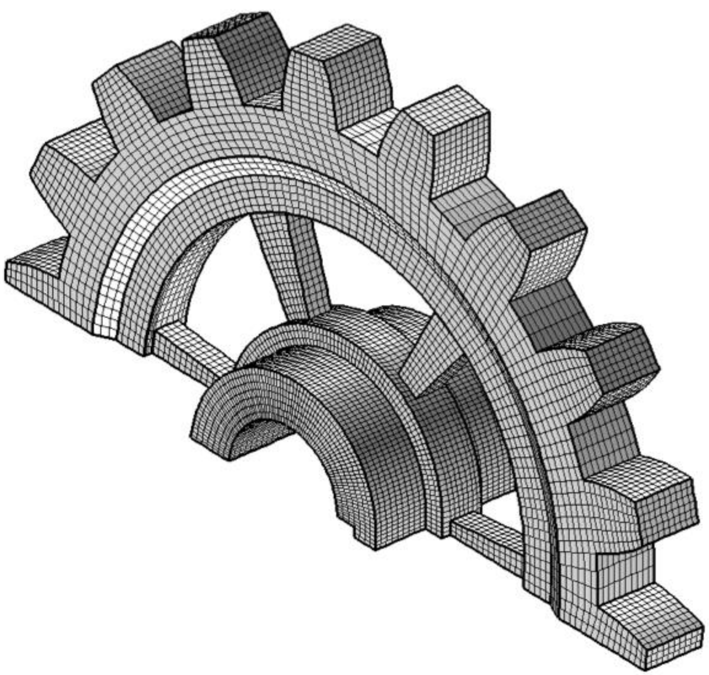
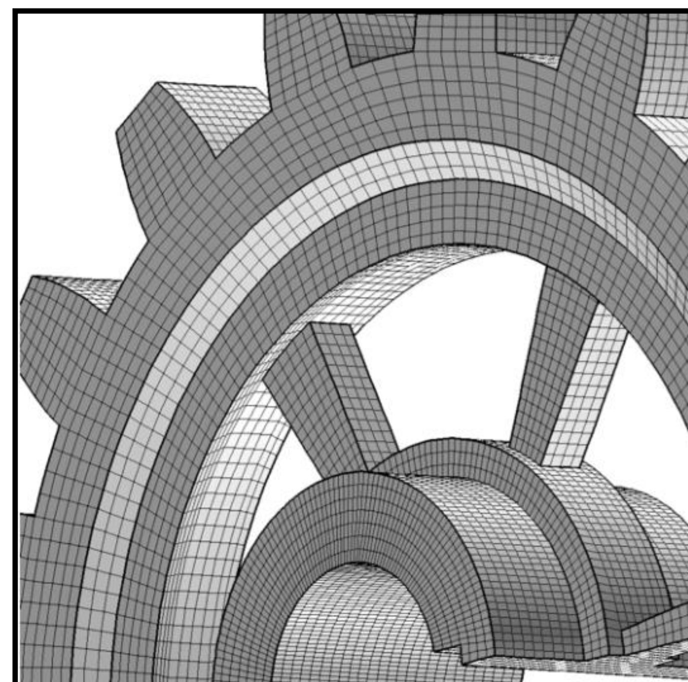
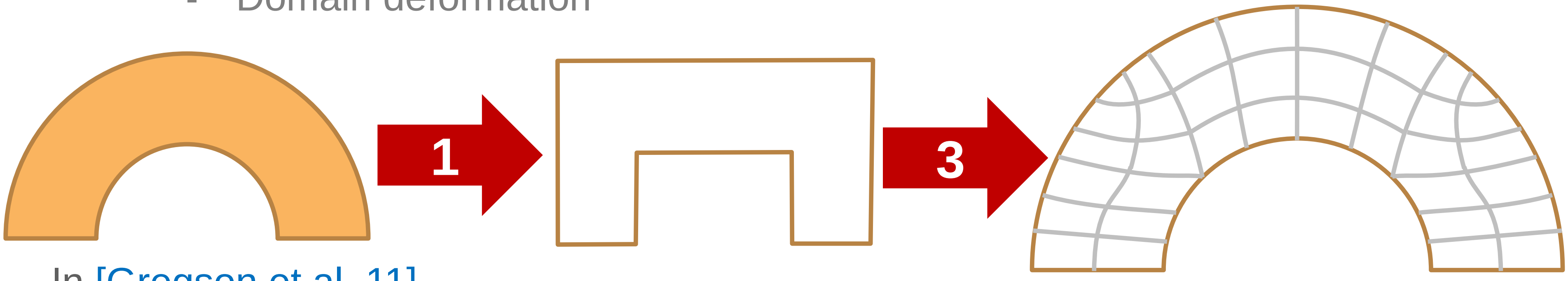




# CARTESIAN IDEALIZATION – MAIN PRINCIPLE

- Polycube-based approaches [Gregson et al. 11][Huang et al. 14]
  - Domain deformation




In [Gregson et al. 11]

**Step 1 –** Iterative process to generate a polycube $\mathbf{P}_{\Omega}$ of $\Omega$

**Step 3 –** Mesh projection from $\mathbf{P}_{\Omega}$ to $\Omega$

| | |
|---|---|
| Structured | |
| Boundary alignment | S P |
| Element size handling | |
| Industrial maturity | |
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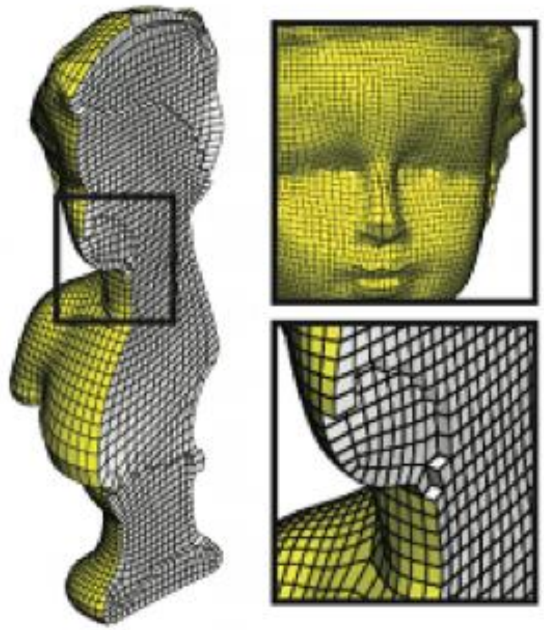
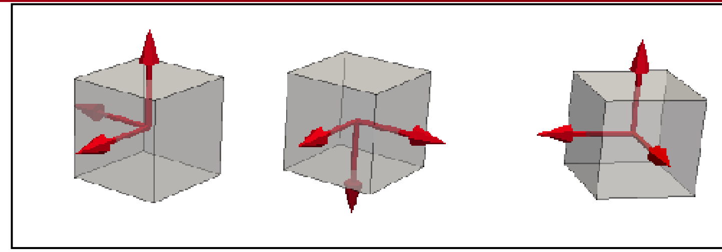
$\mathbf{P}_{\Omega}$

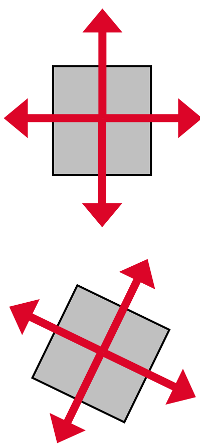


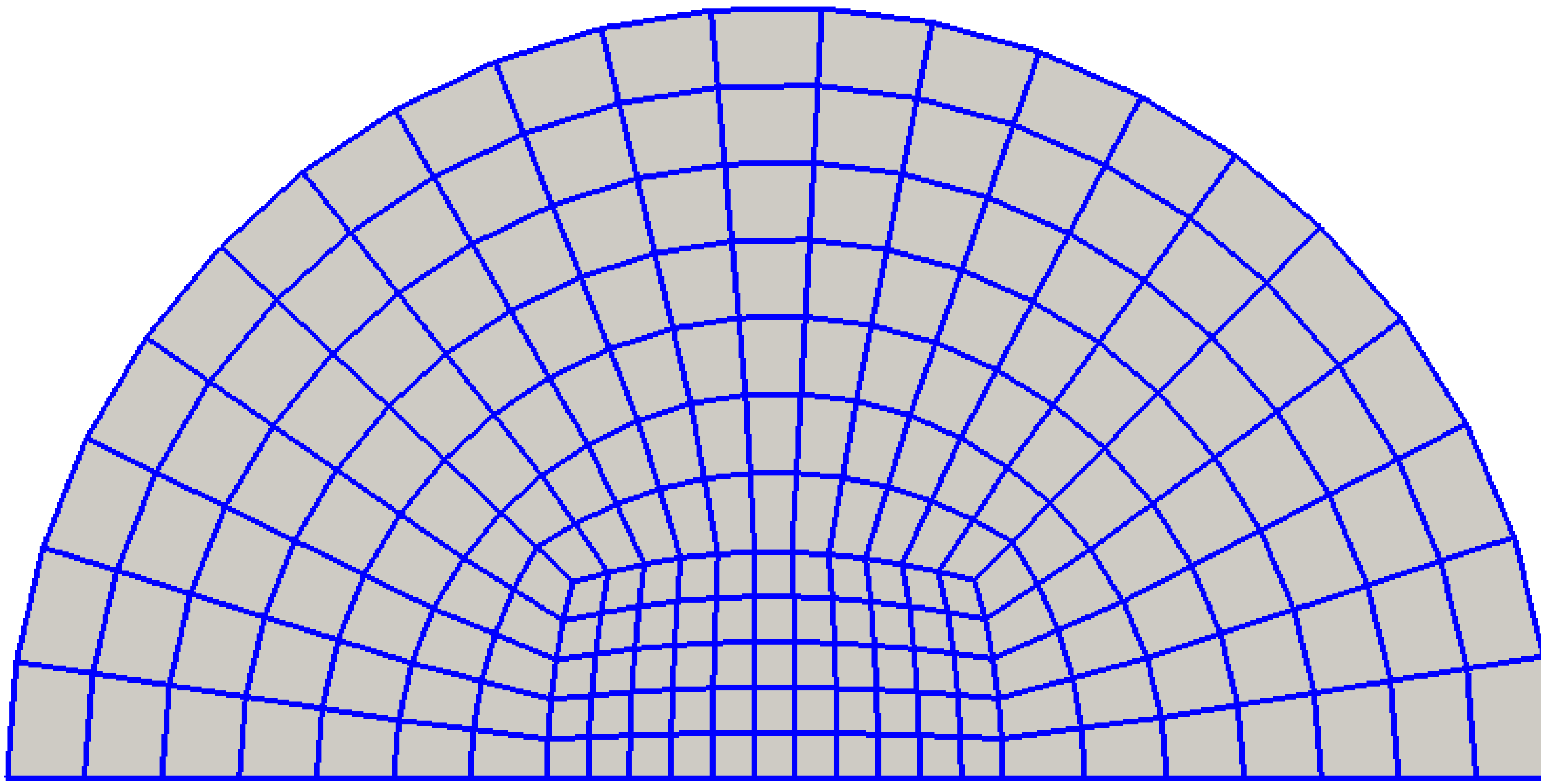
# FRAME FIELDS

## Principle



- Generate a frame field on the domain which provides geometrical data inside the volume
- Naturally boundary-aligned for quad/hex meshes
- With a global structure → smooth transition between elements





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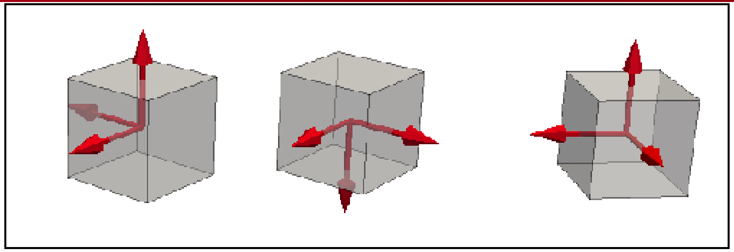

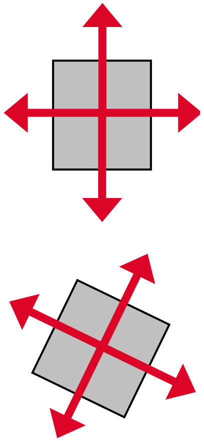

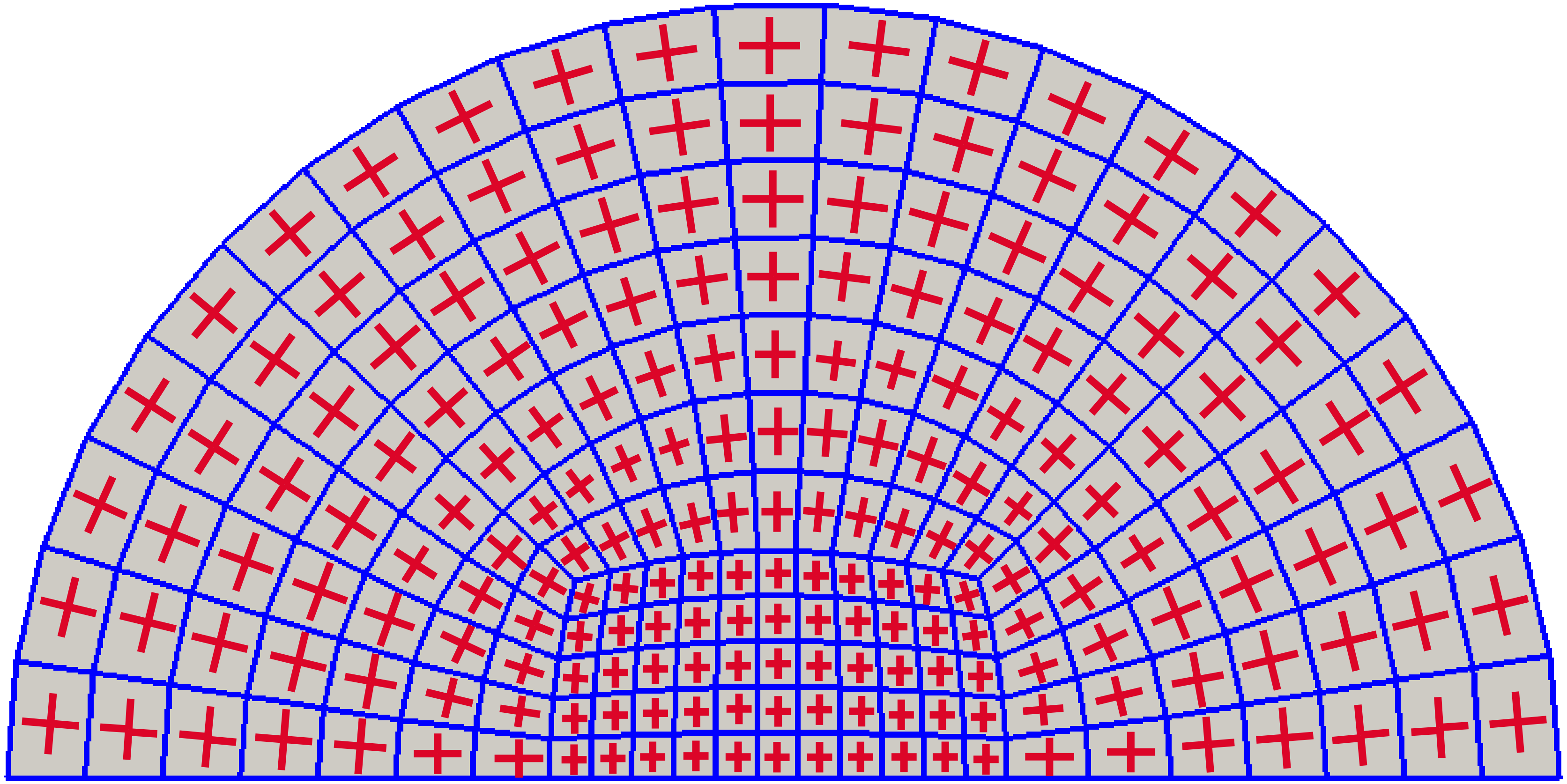

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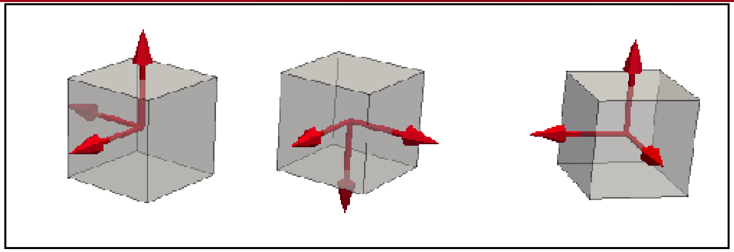

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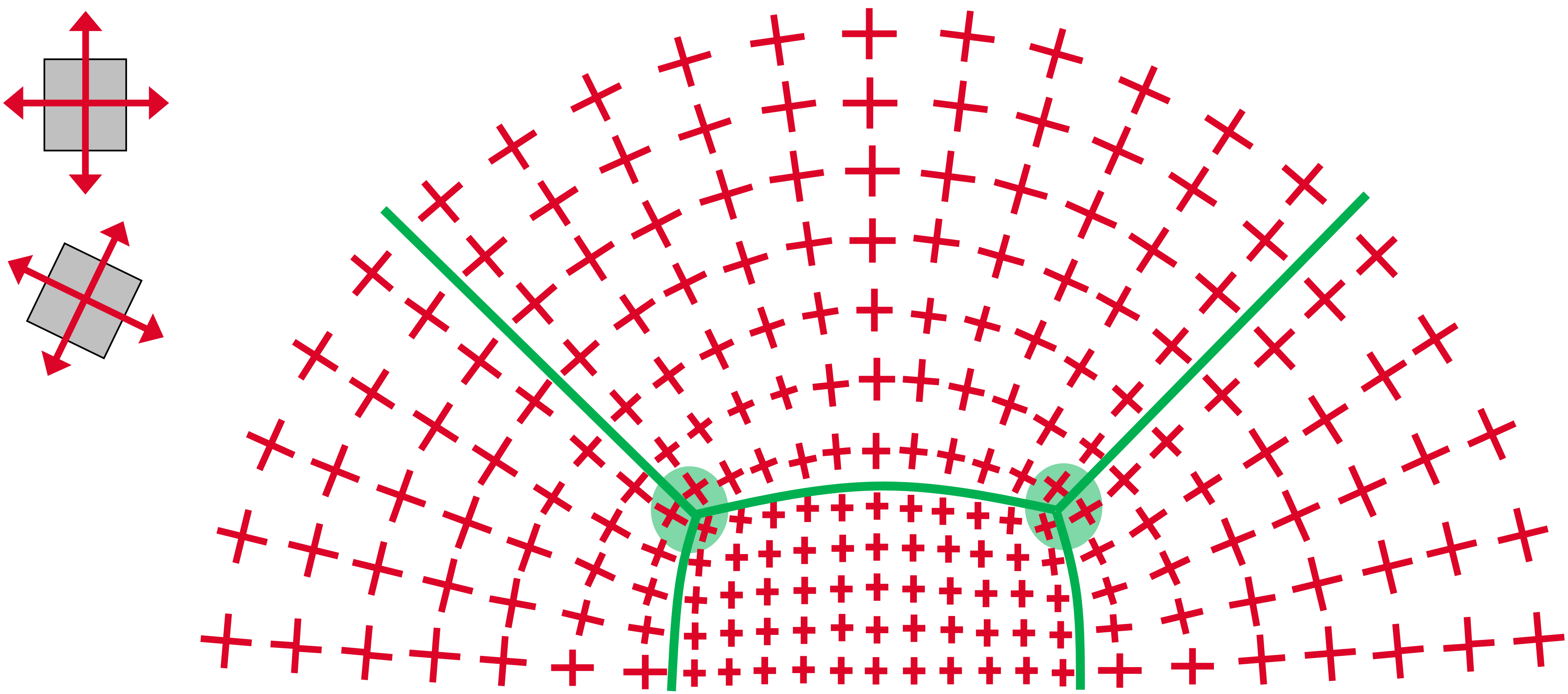
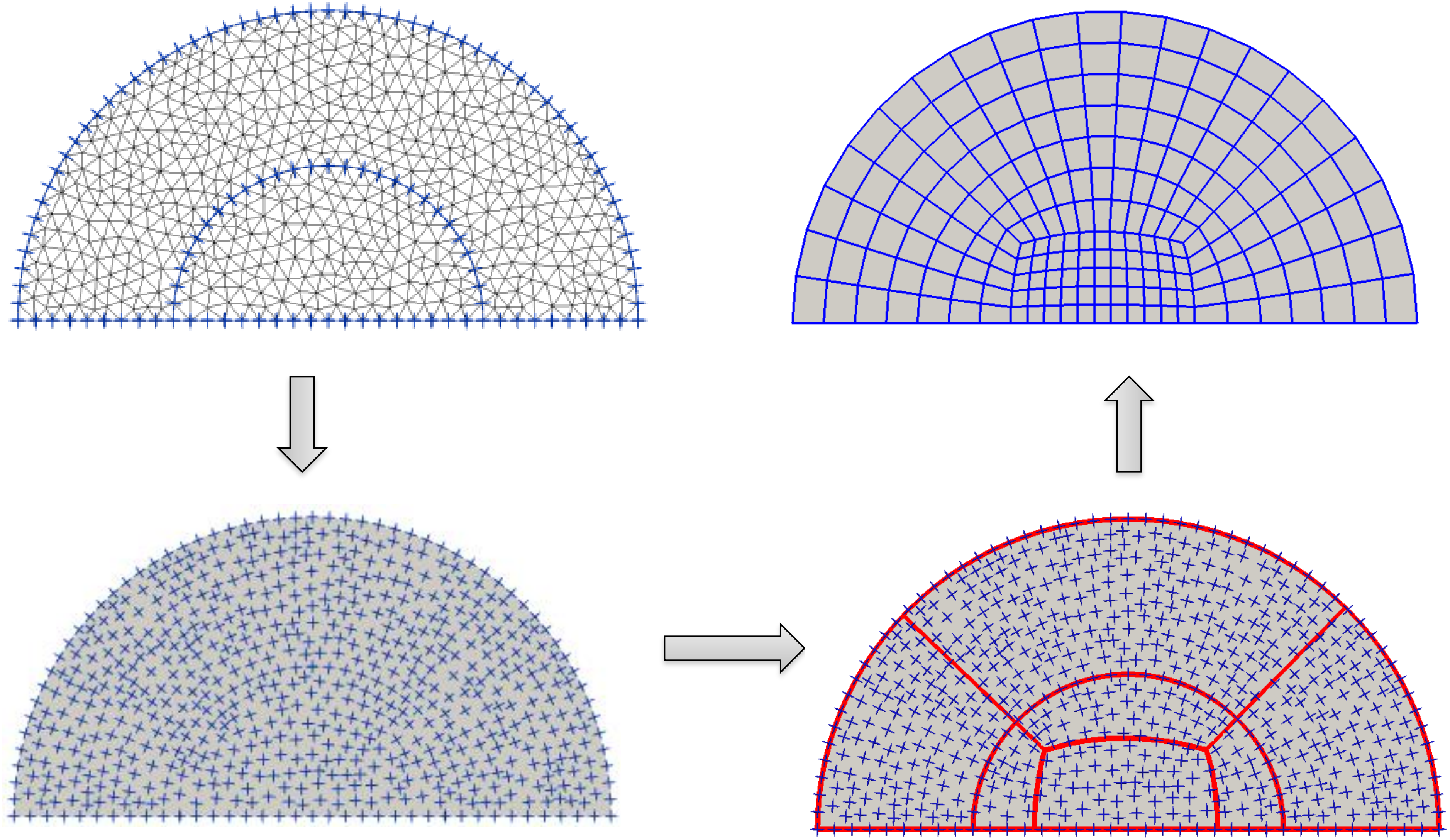
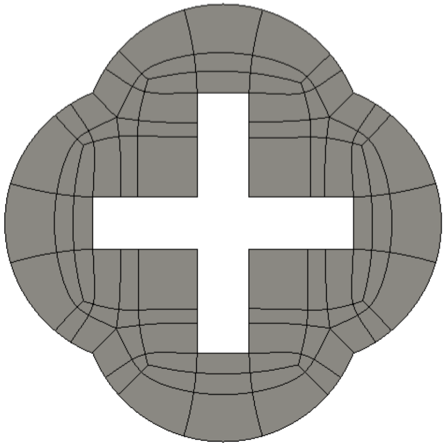


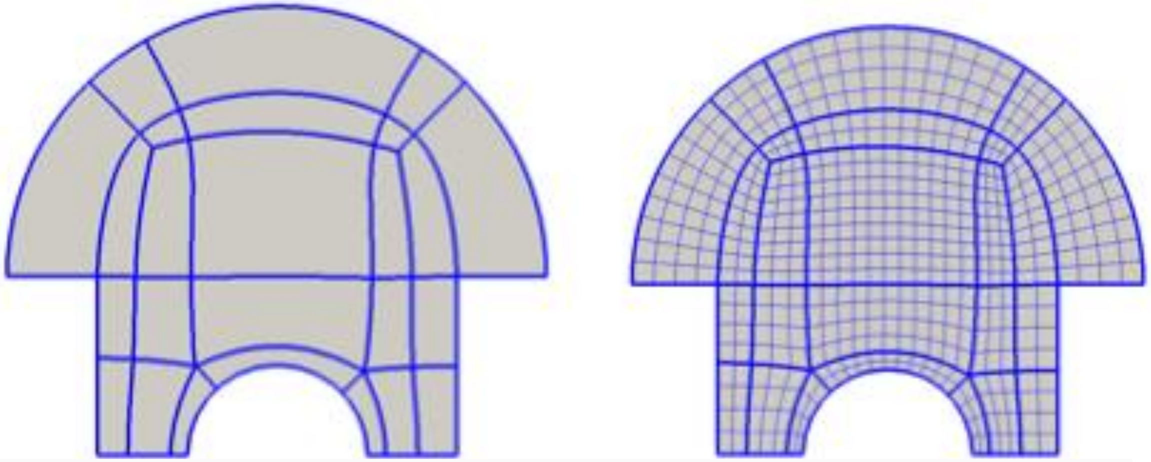
# 2D Frame field Generation

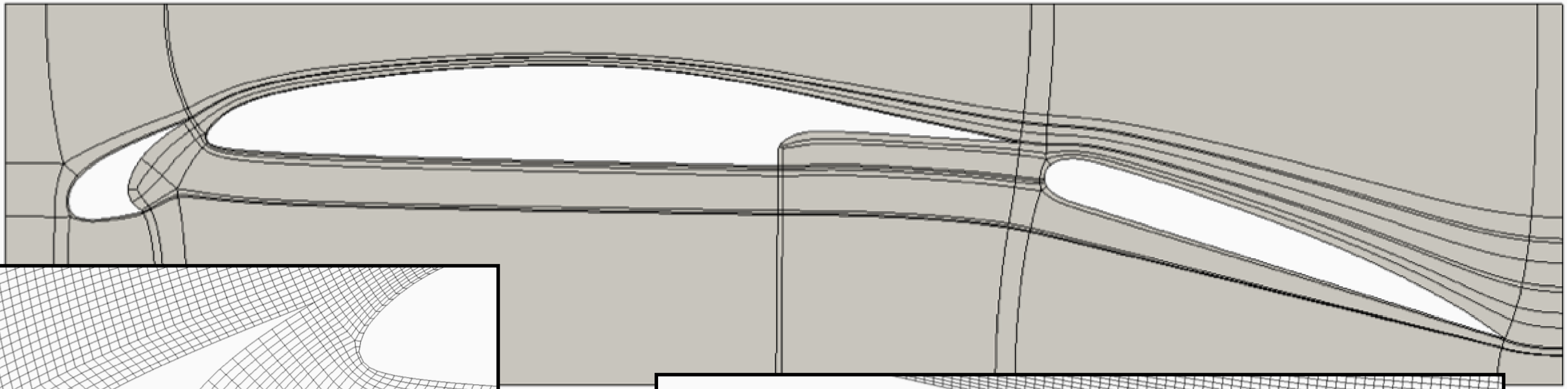


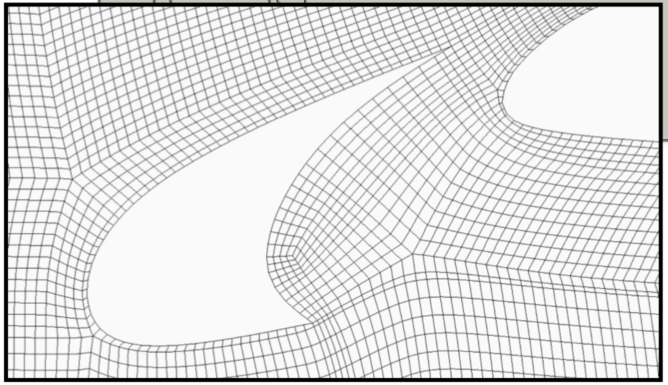
[Kowalski et al. 12] [Fogg and Amstrong 13]

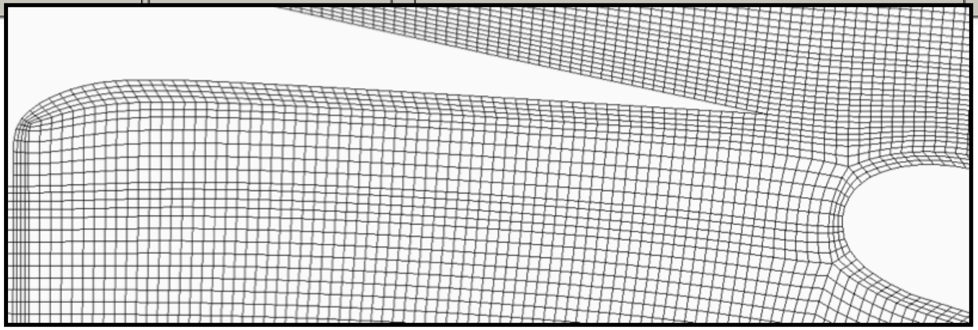
# Frame Field usage – 2D Examples











[Kowalski et al. 12]

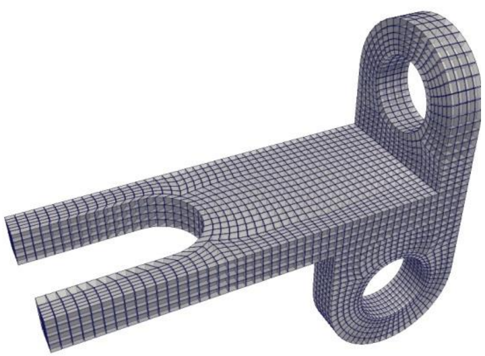
# Frame Field usage – 3D Examples

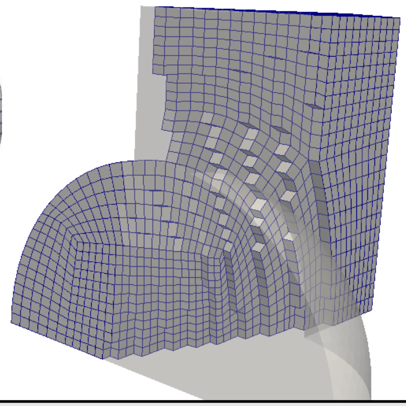
[Kowalski et al. 14]

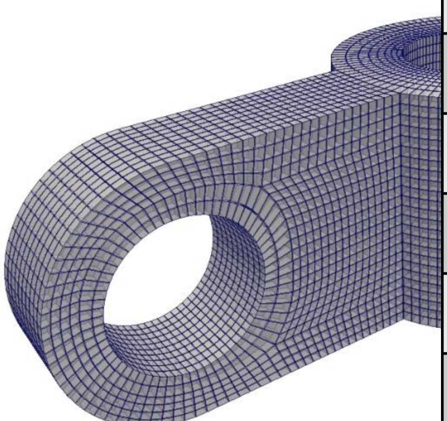
- Generation from a geometric domain
- Block structure extraction
- Vertex-based numerical schema

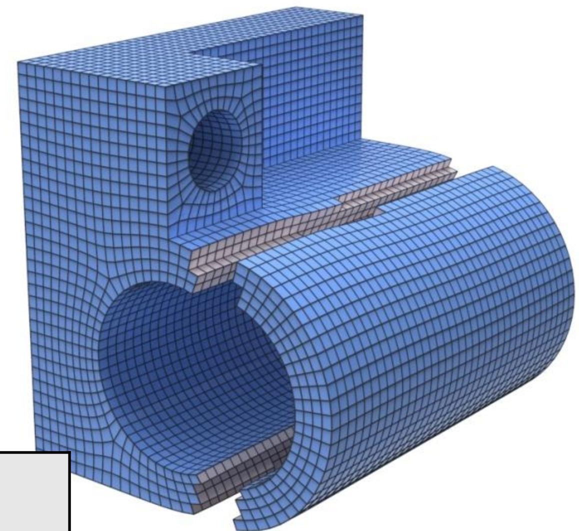
[Huang et al. 11] [Li et al. 12]

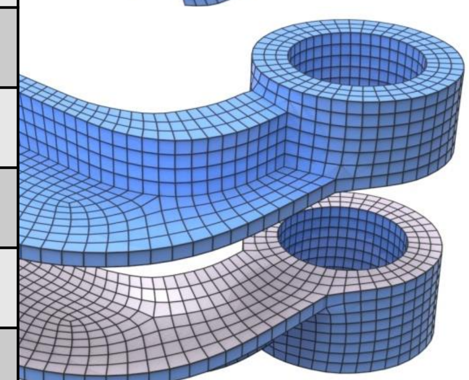
- Generation from a pre-meshed boundary
- Definition of an atlas of parameterization
- Cell-based numerical schema











| | |
|---|---|
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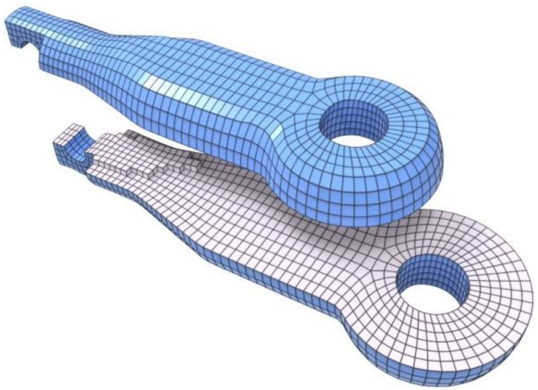
# CONCLUDING REMARKS

## Summary of the different approaches



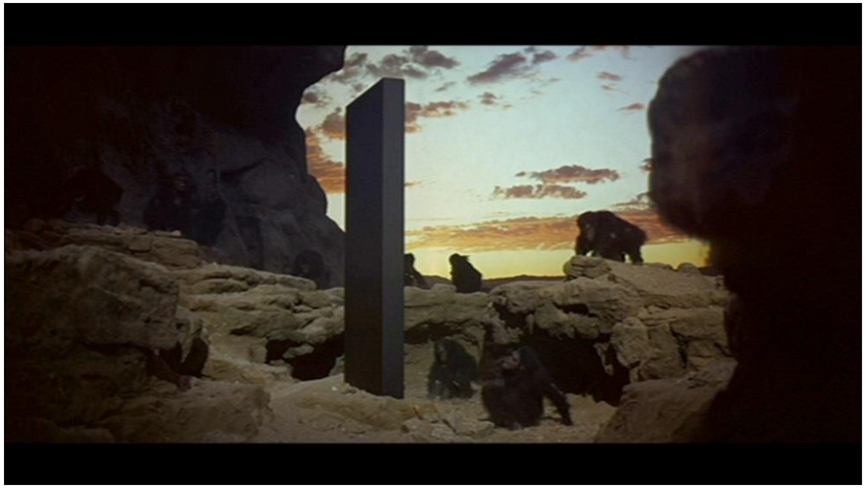
| | Any type of geometries | Respect of the boundary mesh | Boundary alignment | Orientation insensitive | Structured | Element size handling | Industrial maturity | Promising research direction |
|---|---|---|---|---|---|---|---|---|
| Sweeping | NO | NO | YES | YES | It depends | YES | YES | NO |
| Geometric adv.-front | YES | YES | YES | YES | NO | YES | NO | NO |
| Topological adv.-front | YES | YES | YES | YES | NO | NO | NO | NO |
| Overlay-gird | YES | NO | It depends | NO | It depends | YES | YES | NO |
| Medial axis | YES | NO | YES | YES | YES | NO | It depends | It depends |
| Cartesian idealization (Submapping + Polycube) | YES | NO | YES / It depends | NO | YES | NO | NO | YES |
| Frame fields | YES | It depends | YES | YES | YES | NO | NO | YES |

■ YES ■ NO ■ It depends

# TET VERSUS HEX

Oblivion : The Tet

2001 : The Monolith



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