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Incremental progress towards hexahedral mesh generation

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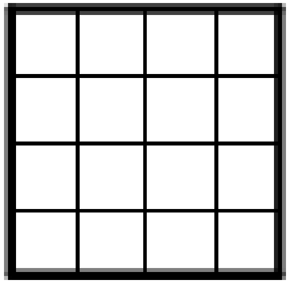
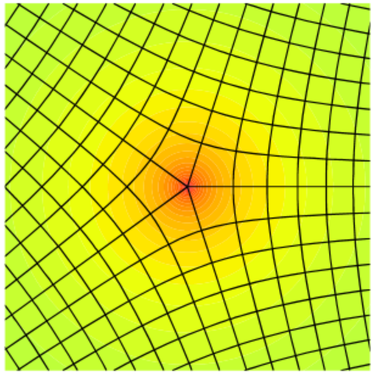
2D mesh singularity points

- Structured quad mesh
  - Four elements meeting at a internal node
  - Grid topology + boundary alignment → too restrictive

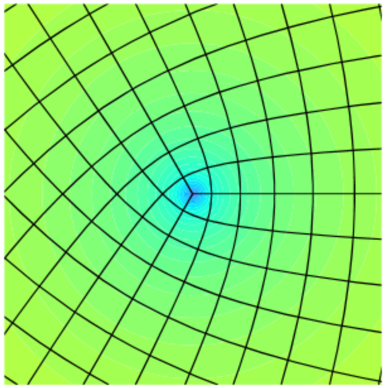


Structured quad mesh

- Block-structured quad mesh
  - Simple blocks meeting at nodes of irregular connectivity, i.e. mesh singularities
  - Positive singularity: > 4 elements, negative singularity: <4 elements



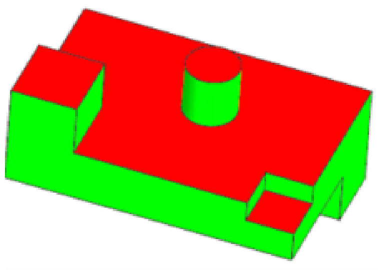
Positive singularity

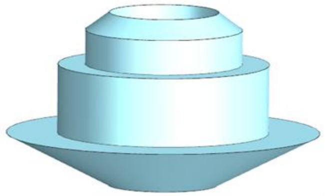


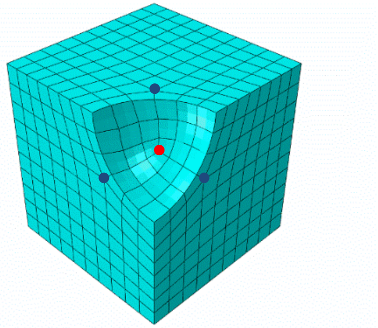
Negative singularity

3D line singularities

- Singularities travels from one face to another
  - Sweeps, multi-sweeps, thin sheets, long slender regions
- Singularities forms loops
  - Revolves
- Singularities meet
  - Limited number of patterns[1]







[1] Price, M. A., Armstrong, C. G., & Sabin, M. A. (1995). Hexahedral mesh generation by medial surface subdivision: Part I. Solids with convex edges. *International Journal for Numerical Methods in Engineering*, *38*(19), 3335–3359.

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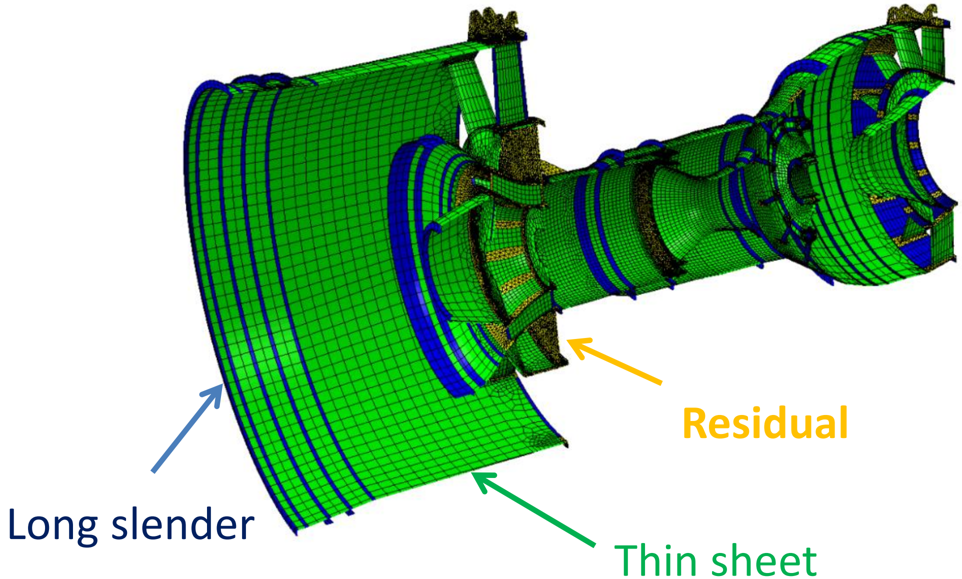
Volume decomposition

- Strategy
  - Thin sheet, long slender and residual regions
    - Reduce the decomposition effort
    - Reduce the DOF of the analysis model



J. E. Makem, C. G. Armstrong, and T. T. Robinson, "Automatic decomposition and efficient semi-structured meshing of complex solids," *Eng. Comput.*, vol. 30, no. 3, pp. 345–361, Dec. 2012.

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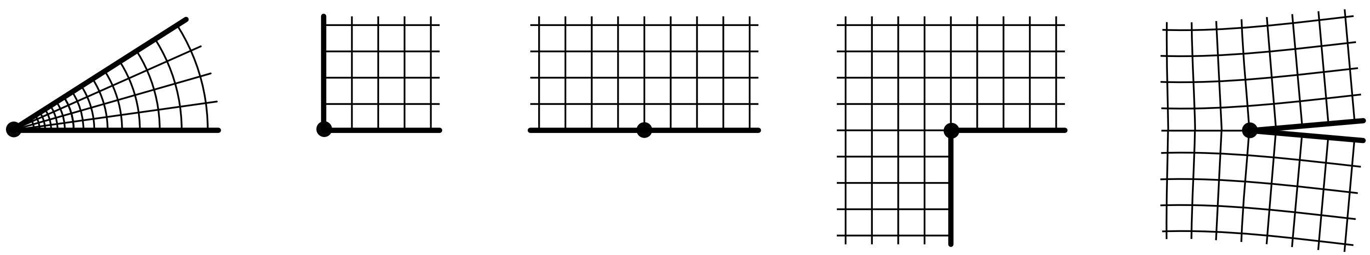
Residual complex regions - mesh patterns at vertices

- Optimum element number n_i at corner angle θ_i

$$n_i = round\left(\frac{\theta_i}{\pi/2}\right)$$



n_c:	0	1	2	3	4
Optimum θ_i range:	$[0, \frac{\pi}{4})$	$[\frac{\pi}{4}, \frac{3\pi}{4})$	$[\frac{3\pi}{4}, \frac{5\pi}{4})$	$[\frac{5\pi}{4}, \frac{7\pi}{4})$	$[\frac{7\pi}{4}, 2\pi]$

Identifying surface singularities

$$\sum_{vertices}\left(\frac{\pi}{2}(n_i-2)+\alpha_i\right)+\sum_{edges}\int \kappa_g\,ds+\iint_{face} K\,dS+(n_+-n_-)\frac{\pi}{2}=0$$

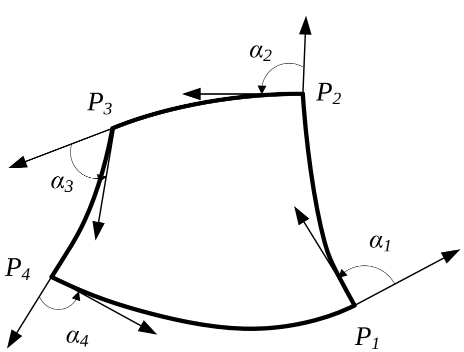
A continuum theory for unstructured mesh generation in two dimensions, G Bunin, CAGD, vol. 25, 14–40, 2008

Euler Characteristic $\chi = V - E + F$

where V, E, F are the number of vertices, edges and faces of any subdivision of the surface

Gauss-Bonnet theorem

$$\oint_{\partial R} k_g\,ds+\iint_R K\,dS+\sum_{i=1}^{N}\alpha_i = 2\pi\chi$$

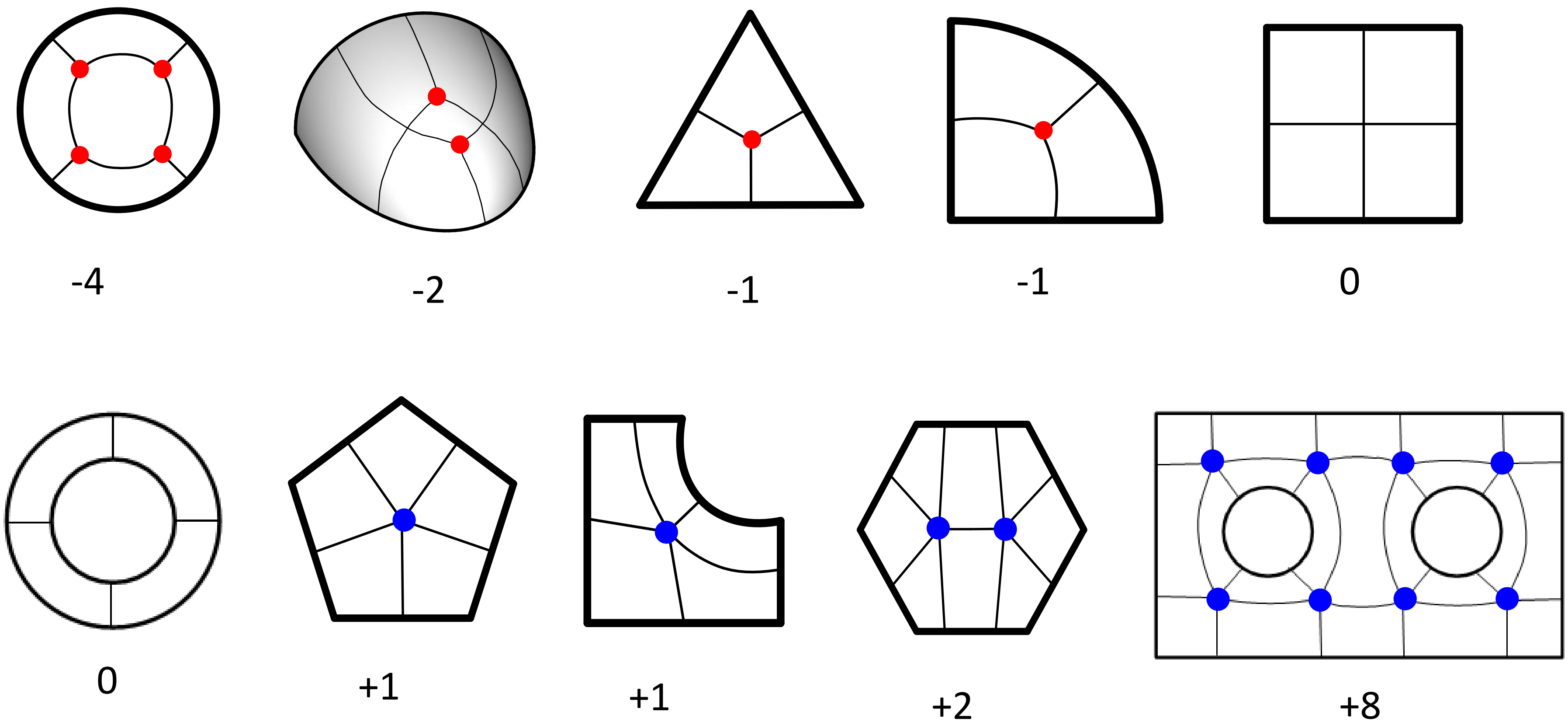


N: the number of corners
n_i: no of elements at each corner

$$n_+-n_- = -4\chi+\sum_{i=1}^{N}(2-n_i)$$

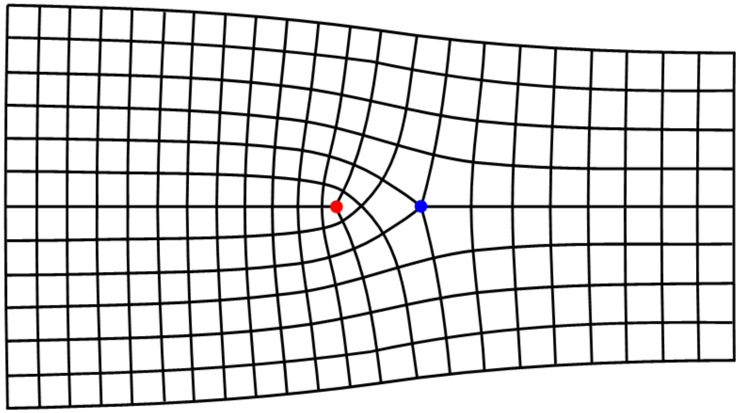
Singularities on simple surfaces

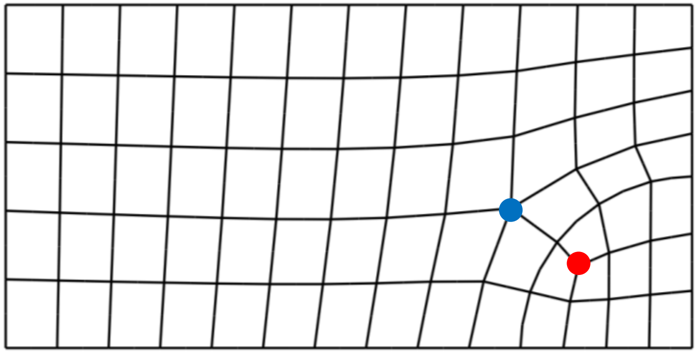
$$n_+ - n_- = -4\chi + \sum_{i=1}^{N}(2 - n_i)$$



Controlling Mesh Density

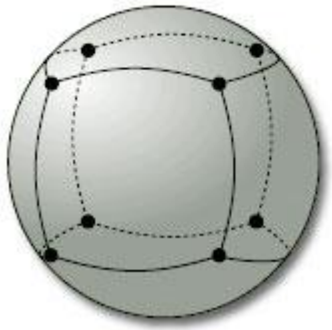
- Extra singularity pair (a dislocation)



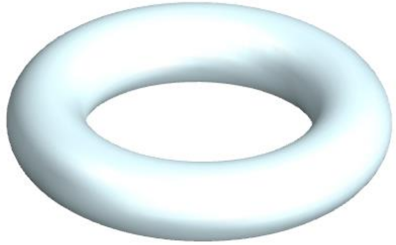


Finding required number of surface singularities

$$n_+ - n_- = -4\chi + \sum_{i=1}^{N}(2 - n_i)$$



χ=2, N=0, n_+-n_-=-8



χ=0, N=0, n_+-n_-=0

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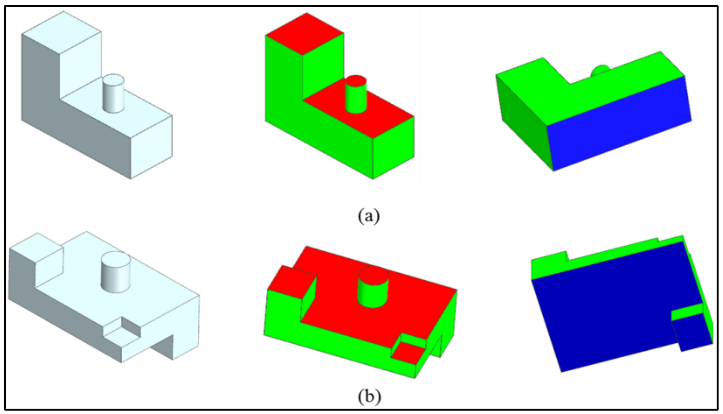
Applications

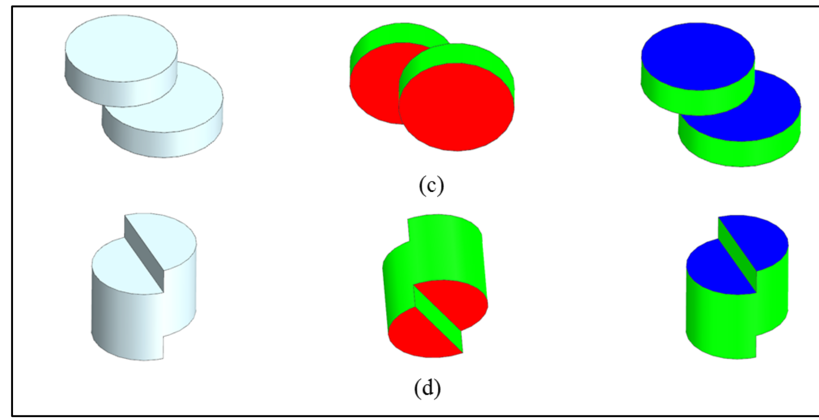
- Identify sweep-able volumes [1]
  - No mesh singularities on wall faces

Wall faces

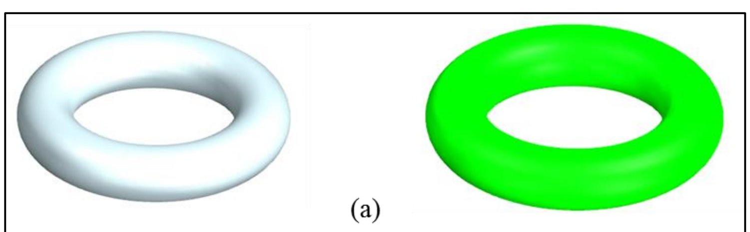
Source faces

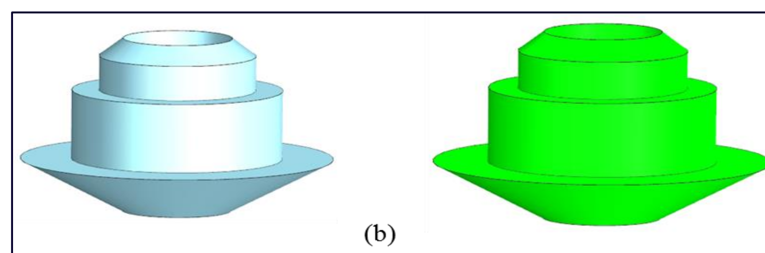
Target faces


(a)
(b)


(c)
(d)

- Revolves


(a)


(b)

[1] D. R. White and T. J. Tautges, "Automatic scheme selection for toolkit hex meshing," Int. J. Numer. Methods Eng., vol. 49, no. 1, pp. 127–144, 2000

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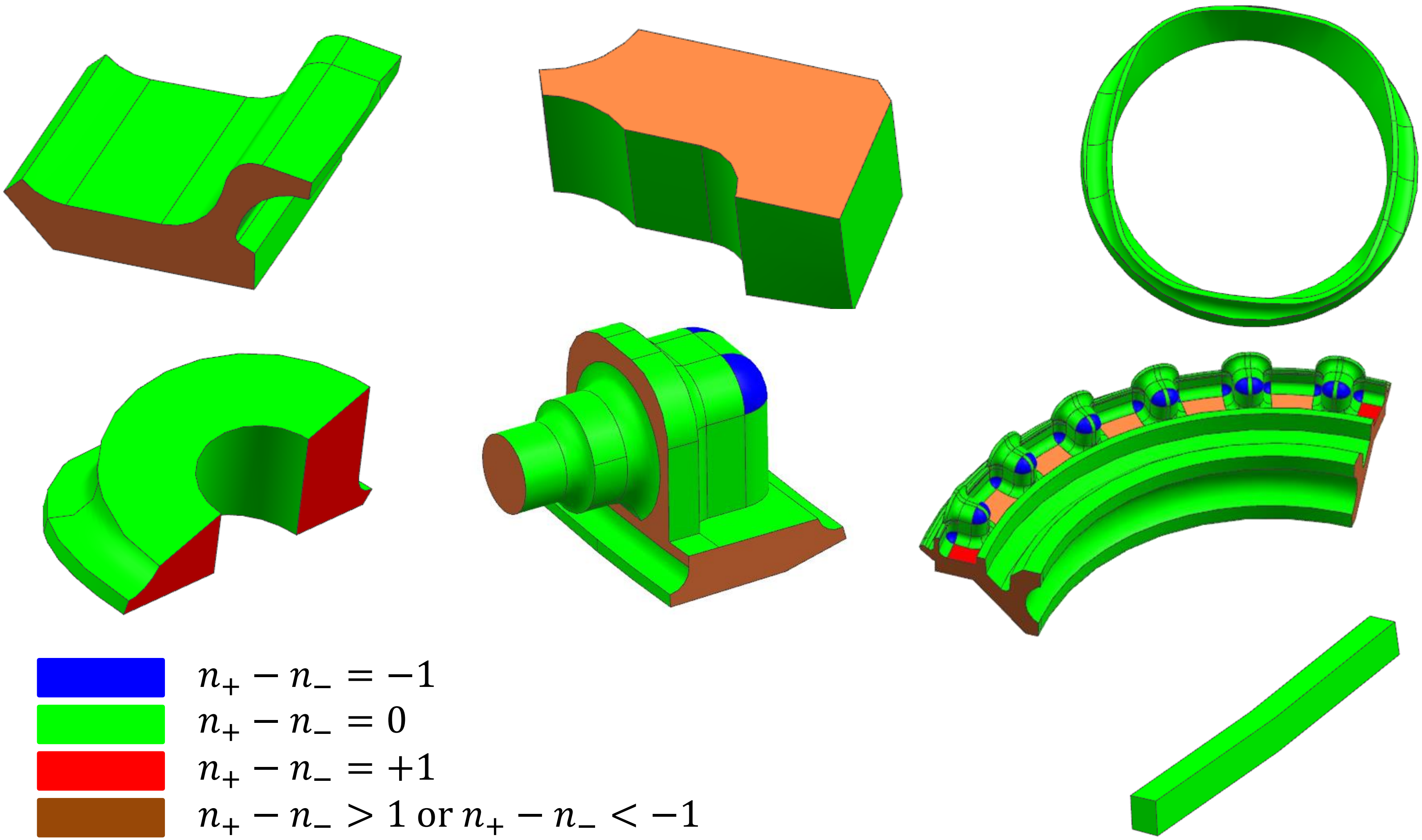
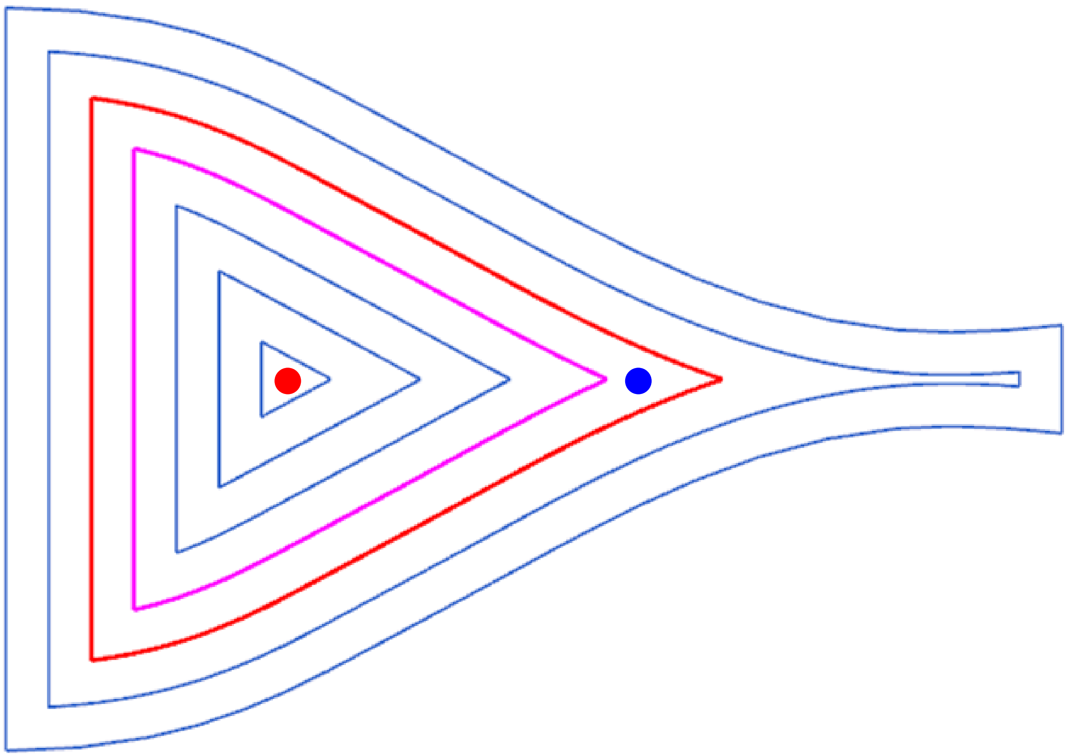
Surface singularities in the 3D residual regions



Singularity placement: using offsets

- Locate the position of the singularities
  - When the number of singularities changes after offset, a singularity should be placed



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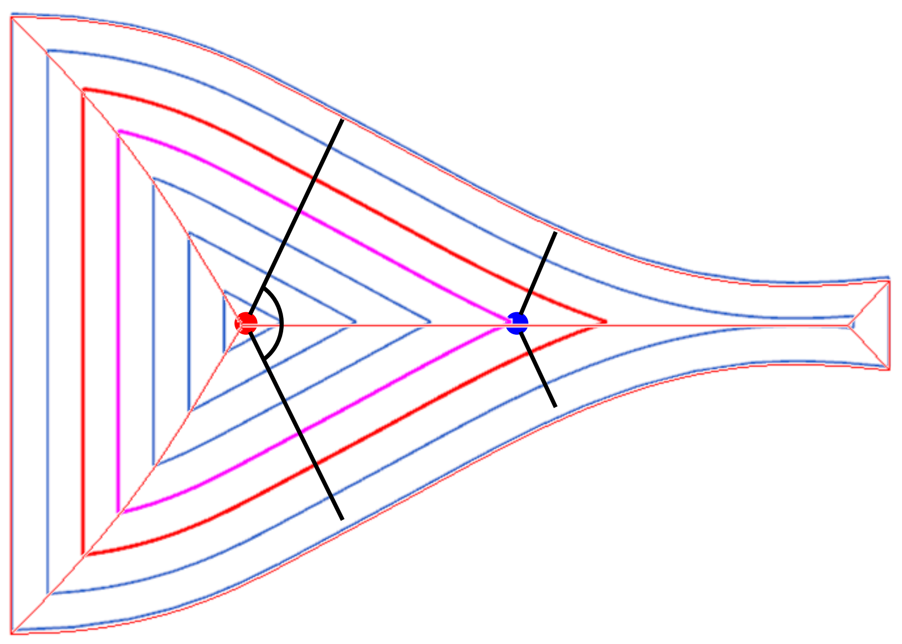
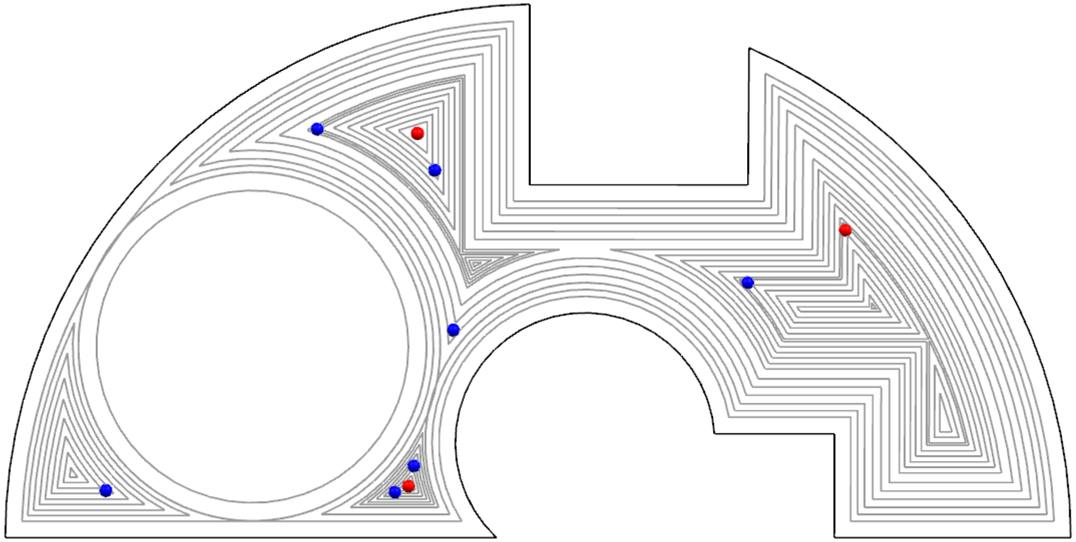
Singularity placement: offsets vs medial axis[1]

- Locate the position of the singularities
  - When the included angle between medial radii changes

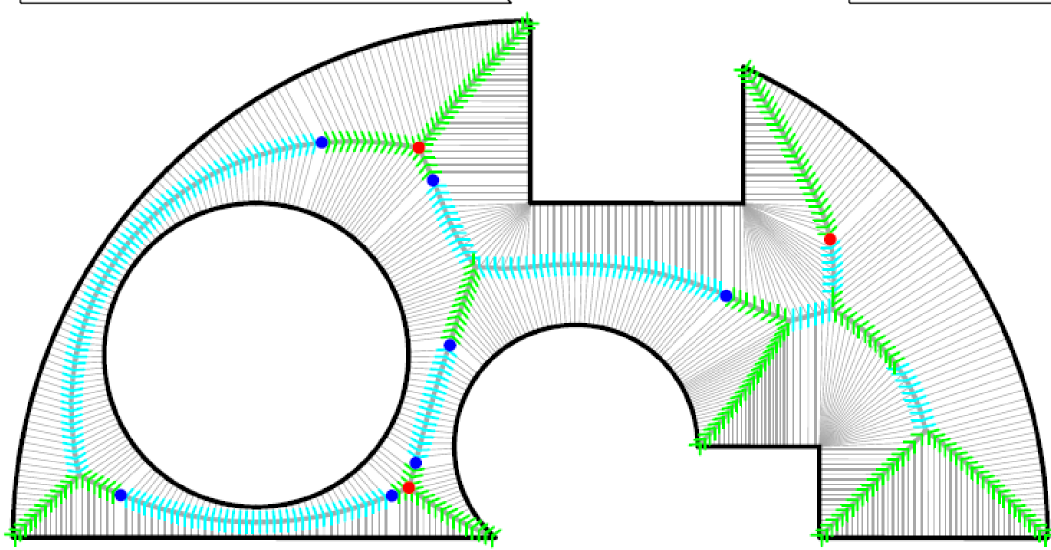


[1]Harold J. Fogg, Cecil G. Armstrong, and Trevor T. Robinson. "Enhanced medial-axis-based block-structured meshing in 2-D." CAD 72 (2016): 87-101.

Singularity placement: medial axis vs offset



Singularities calculated by making offset of the boundary



Singularities calculated from medial axis

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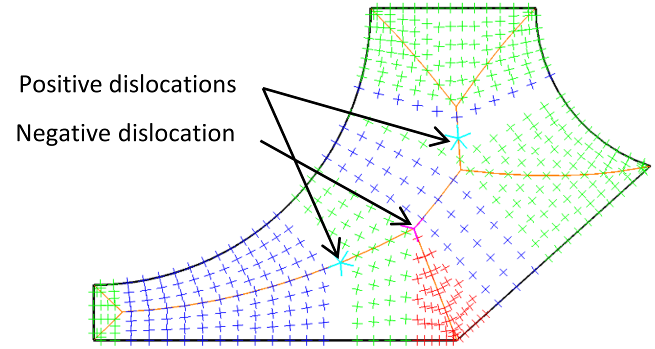
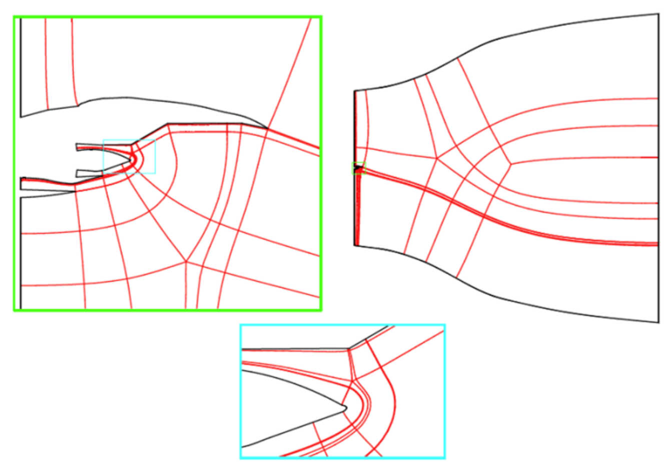
Singularity placement: medial axis vs cross field

- Advancing front of crosses
- Singularities occur where MAT changes from aligned with mesh to diagonal
- Can handle
  - Variations in target element size, shape and orientation
  - Large differences in feature size
- Doesn't need precise MAT, but singularities end up in very similar places for isotropic elements





N. Kowalski, F. Ledoux, and P. Frey, "Automatic domain partitioning for quadrilateral meshing with line constraints," *Eng. Comput.*, pp. 1–17, 2014.
H.J Fogg, C.G. Armstrong, T.T. Robinson, "Automatic generation of multiblock decompositions of surfaces," IJNME, 101, No. 13, pp. 695-991, 2015.

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Conclusions

- Placement of mesh singularities is key to structured multi-block hex meshing
- Thin sheets: singularities start on one surface and exit on the opposite one
- Long slender regions: singularities run from source to target faces
- Multi-sweep regions: similar
- Revolves: singularities form a loop
- Residual complex regions
  - Simple analysis using Euler characteristic and number of elements at each corner provides minimum necessary number of singularities emerging on each face
  - Can add addition positive/negative singularity pairs to provide target mesh size distribution
- An incremental approach helps identify strategies for different singularity patterns

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Acknowledgements

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